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Robustness of regional matching scheme over global matching scheme

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Abstract

Analyzing the effect of concentrated noise on a typical decision-making process of a simplified two-candidate voting model, we have demonstrated that a local approach using a regional matching process is more robust and stable than a direct approach using a global matching process, by establishing that the former is capable of accommodating a higher level of noise than the latter before the result of the decision overturns. To extend the theory to imagery analysis, we pose a conjecture that our conclusion on the robustness of the regional matching processes remains valid not only for the simpler vote counting schemes but also for practically more important decision-making schemes in image analysis which involve dimension-reducing transforms or other features extraction processes such as principal component analysis or Gablor transforms. Two convincing experimental verifications are provided, supporting not only the theory by a white-black flag recognition problem on a pixel-by-pixel basis, but also the validity of the conjecture by a facial recognition problem in the presence of localized noise typically represented by clutter or occlusion in imagery.

Keywords: Stability; Voting; Global matching; Regional matching; Noise; Pattern recognition; Decision making

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[1] The major work of this paper was done at the time when this author was with Sunflare company.

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1. Introduction

In dealing with many image analysis or pattern recognition problems, image specialists have quite intuitively resorted to local descriptions rather than holistic features. For example, regional Gabor transforms [5] or wavelet methods are extensively used in preference to global Fourier transforms in image and pattern analysis. The purpose of this paper is to elucidate and to support this strategical preference on a general basis by providing a concrete and quantitative model for establishing a mathematical foundation. We show this by demonstrating that the local approach of a regional matching process is more stable than the holistic approach of a global matching process against concentrated types of noise.

Consider a decision-making process G, which arises quite frequently in much scientific research as well as in daily life. Given several candidates (or selections) to choose from, we must choose one candidate (selection) by using a voting scheme, or by matching the features extracted from the entire area (or the nation). We call G a global voting (or matching) method. The global method is extensively used in electing not only heads of local governments such as governors or mayors, but also presidents of many countries, such as France and Peru. Now let us convert G to a new version $r(G)$, where we decide the winner of each pre-divided region by G's, but we make the final decision by a simple majority of the winning regions using the “winner-take-all" principle within the pre-divided regions. A most well known case of the converted version $r(G)$, called regional voting (matching) method, is the US presidential election system. It is the robustness of decision making by the G or the $r(G)$ process against a concentrated type of noise that we seek to clarify in the present analysis. Here, G could be any decision-making procedure. In a simple voting system, G makes use of voting and makes a decision based on the majority of votes counted. In facial recognition, G based on this voting system may involve pixel-by-pixel comparison between two facial images or any of various features extraction schemes, including the widely used dimension-reducing schemes, such as principal component analysis (PCA) [4] and Gabor transform [5], the former PCA scheme leading to the famous eigenface method [7]. The resulting reduced facial space of PCA is instrumental in obtaining the major eigenfaces of the training set, for example. It is easy to find the co-ordinates of a new face projected into the facial space, and then to make a recognition decision by matching the projection with stored images of models. The corresponding $r(G)$ could be implemented by first dividing the whole two-dimensional rectangular image frame into smaller regions (of equal size in our analysis). Within each region, we make use of the PCA method in choosing a winner, and we make the final decision for the whole image by a simple majority of the winning regions where the winner gets all the votes of the region. It is the improved stability of $r(G)$ over that of G that has motivated our current research, as verified by the several numerical examples given in this paper. The result that we will derive formally in this paper is intuitively clear. We ask which of the two methods is the more stable against an external source of concentrated noise. Evidently a higher level of noise is needed to overturn the decision of the regional matching scheme, because while every piece of noise added in the global matching scheme contributes evenly to the final reversal of a decision, most pieces of localized noise added in the regional matching are used up in affecting only a small fraction of all of the divided regions; as a result, a

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considerable amount of noise added will be used up within those restricted regions, leaving many of the regions undisturbed.

We adopt the simplest possible two-candidate voting model in our analysis, in which each cell in the nation can cast only one vote, although an extension to a multi-candidate model is given in Section 3.2.1. Thus G itself can be regarded as a simple national voting scheme, where the winner is decided by a simple majority of votes. To simplify the analysis on regional matching, we divide the nation into smaller regions of equal size, where the winner is decided by the number of winning regions, the winner of a region being decided by G. We set up a noise-and-voting model for this simple situation and show that when the size of the regions is reasonably small, the regional voting scheme is more stable than the national voting scheme. A conjecture is posed that this model is valid in a more general decision-making process, where G involves a decision-making process by PCA matching or Gabor matching. We present an experimental verification to support the conjecture in Appendix A.

The present paper is constructed as follows: In Section 2, we first give precise definitions of noise, noise-concentrated area, and the number of noise-contaminated regions, including basic assumptions used in the analysis. We prove Theorem 2.1, which relates the noise-concentrated area and the number of potential noise-contaminated regions. Theorem 2.2 on Shifting Strategy shows how we can improve the relation. The resulting Corollaries 2.1 and 2.2, corresponding to Theorems 2.1 and 2.2, respectively, give lower bounds on the noise level to a breakdown point of decision beyond which the decision of voting may overturn. In Section 3, we examine the results of Section 2 from various angles. A convincing experimental verification of the theory is presented in Section 4, using a black-and-white flag recognition problem confirming the validity of the theory on a pixel-by-pixel basis. An experimental verification given in the appendix supports the conjecture that the theory developed for the pixel-by-pixel voting process remains valid for more general decision-making processes involving dimension-reducing schemes, such as PCA and Gabor transform.

2. Theorems

2.1. Notations

The key notations and basic assumptions used in the paper are summarized here.

We suppose that a nation (or an entire image for an image application) consists of N unit cells (or pixels), each having one vote to exercise; for simplicity, a nation is always represented by a rectangle of size $l \times m$, so that $N = l \times m$. A nation on the other hand can be partitioned into K equal sized, square regions of $m_r \times m_r$. We also assume that both l and m are divisible by m_r and that the pair of the opposing edges along the outer boundary of the rectangular nation are to glide onto the other end as glued together so that the nation can have a total of $m_r^2 = N/K$ different partitions. Like m_r, which is the length scale of a square region, m_n denotes that of a noise-concentrated block, with the terms "noise" and "noise-concentrated block" being defined in Definition 2.2 of the next subsection.

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2.2. Main theorems

We analyze in this paper a simplified model allowing only two candidates, A and B, in the election. Without losing generality, we assume that, in the absence of external sources of noise, $A\%$ of the total cells vote for A, and $B\%$ cells vote for B, so that

$$A\% + B\% = 1 \quad \text{and} \quad A\% > B\%.$$

We discuss our possible extension to an n-candidate system in Section 3.2.1. We examine the effect of "concentrated" noise on election results, based on the decision-making processes of global voting and regional voting. In image processing applications, that type of noise is often observed in imagery containing transparency, specular reflections, shadows, or fragmented occlusion as seen through the branches of a tree or a sun shade [2].

The formal definitions of concentrated noise, global voting, and regional voting are given below:

Definition 2.1 (*Voting*).

- *National Voting*—The entire population N of a nation vote either for candidate A or B, and candidate A wins *if and only if* he gets a majority of the N votes.
- *Regional Voting*—The population m_r^2 ($= N/K$ for K regions) of a region vote for candidate A or B; a majority of votes determine the winning candidate for the region; and a majority of the K winning regions, not necessarily the majority of the entire population N of the nation, determines the winner for the nation.

Definition 2.2 (*Noise*).

- *Anti-A-Noise*—A set of noise is called *anti-A-noise* (or *anti-B-noise*) if all the cells under influence vote for B (or A), regardless of whether each cell originally votes for A or B. The number of the cells under influence is called the number of noise units.
- *Noise-Contaminated Vote*—A vote is called noise-contaminated if the vote of a cell undergoes a change either from candidate A to B or from candidate B to A under some change of environmental conditions. The noise-contaminated vote undergoing a change from candidate A to B (or B to A) is called *anti-A-noise-contaminated vote* (or *anti-B-noise-contaminated vote*), respectively.
- *Anti-A-Noise-Concentrated Block*—An anti-A-noise-concentrated block (anti-B-noise-concentrated blocks) is defined as a non-overlapped $m_n \times m_n$ sized area whose cells are all under influence of anti-A-noise (anti-B-noise).
- *Anti-A-Noise-Concentrated Area*—An anti-A-noise-concentrated (anti-B-noise-concentrated) area is defined as the union of all anti-A-noise-concentrated (anti-B-noise-concentrated) blocks.
- *Anti-A-Noise-Contaminated Region*—A region is defined as anti-A-noise-contaminated (anti-B-noise-contaminated) *if and only if* the conjunction set of the region and the anti-A-noise-concentrated (anti-B-noise-concentrated) area is not empty.

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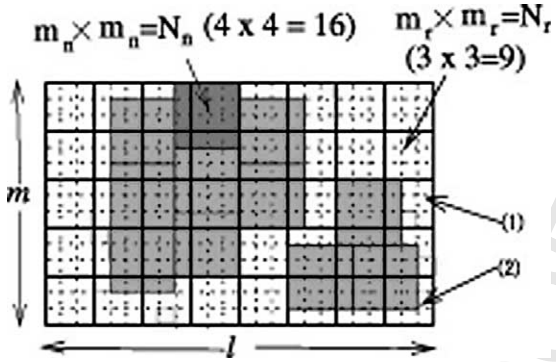

Fig. 1. Noise-concentrated blocks, noise-contaminated regions and noise-concentrated area in a partitioned nation.

Fig. 1 illustrates how noise-concentrated block, noise-contaminated regions and noise-concentrated area as defined here are related. In the figure, the nation is of size $N = l \times m = 24 \times 15 = 360$ cells, the equally subdivided regions are of size 3×3 cells so that the number of regions $K = 40$, while the anti-A noise-concentrated blocks are of size 4×4. The reader may now convince himself that the anti-A-noise-concentrated area as defined above is of size 176 ($= 11 \times 16$) cells, while the total size of anti-A-noise-contaminated regions is 252 ($= 28 \times 9$) cells, typically including regions (1) and (2). Note here that out of the entire 40 regions, 28 regions are anti-A-noise-contaminated. To simplify the calculation of the lower bounds of the noise for regional voting, we will regard that all the anti-A-noise-contaminated regions vote for candidate B even if the number of votes for B within each of the regions may not dominate that for A.

In the analysis, we assume that there is only anti-A-noise.

Assumption 2.1. The effects of anti-B-noise on election results will be ignored in the analysis.

This assumption will be justified for the following two reasons: First, the anti-B-noise and the anti-A-noise are independent, so that we may consider the effect of the anti-A-noise entirely independently of the anti-B-noise. Second, we want to establish a lower bound to a breakdown point in the prevailing situation of $A\% > B\%$. We see that the anti-A-noise gives a lower bound in terms of a noise level up to which we can accommodate before the original result of the regional or global voting decision is reversed. The lower bound for regional voting will be established in Theorems 2.1 and 2.2 and Corollaries 2.1 and 2.2, while an exact bound for global (national) voting is given in Observation 2.1.

As we have emphasized, we consider locally "concentrated" noise alone. Thus we have:

Assumption 2.2. All the anti-A-noise is within an anti-A-noise-concentrated area.

This assumption will always hold, because we can regard the smallest block size of a single cell as the size of a noise-concentrated block in the worst case. For general cases of $m_n \times m_n$ block size in which $m_n > 1$, it is not difficult to conclude that a possible error

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between the anti-A-noise of the nation and the anti-A-noise-concentrated area becomes negligibly small as the size of each of the noise-influenced areas increases sufficiently. Thus this assumption will not affect the validity of any subsequent theorems. We will clarify the situation in Observation 3.2 of Section 3.2.3.

In addition, we make the following assumptions in terms of the definitions we have introduced:

Assumption 2.3.

- *Average Distribution Assumption*—We assume that in the absence of noise, the voting distribution of the undisturbed national voting prevails in *any* sufficiently large size areas, whether consisting of a continuous part of the nation or of randomly chosen blocks of cells.
- *Region Size*—We assume that the size of equally partitioned regions is sufficiently large so that in the absence of noise, the average distribution assumption above holds in each of the partitioned regions.

The assumption implies that, in the absence of noise, the global voting behavior of $A\%$ and $B\%$ prevails in each of the regions such that there are almost $A\%(N/K)$ cells voting for A and $B\%(N/K)$ cells voting for B. This assumption can be relaxed (see Section 3.2.5).

We conclude that, if Candidate A (or B) wins in the national voting, so does Candidate A (or B) in each of the regions and hence in the regional voting.

Observation 2.1. *If there exists more than $\frac{1}{2}(A\% - B\%) \times N$ of anti-A-noise-contaminated votes, that is, if $\frac{A\%-50\%}{A\%}$ of the original votes cast for A should change to B, then the noise is effective in reversing the candidate selection from A to B in the national voting. We say the national voting can accommodate $\frac{1}{2}(A\% - B\%) \times N$ noise before a reversal of the original voting result takes place.*

Definition 2.3. We call a region anti-A-noise-contaminated *if and only if* the conjunction set of the region and the anti-A-noise-concentrated area is not empty.

The following lemma shows that we partition the nation into K regions such that the noise-concentrated blocks are concentrated into some fraction of the K regions, providing a clue why the regional voting is capable of accommodating a higher noise level than the national voting, because only a fraction of the entire regions absorb the dominant effects of the noise imposed.

Lemma 2.1. *For any given small positive integer s, we can always choose a partition of the rectangle nation into K regions such that anti-A-noise is concentrated among a fractional K' regions of the K regions ($K' \leqslant K$) so that the total size of these K' regions is less than that of the anti-A-concentrated area plus s units.*

Proof. The lemma can be proved directly by considering the worst case: namely we can always divide the rectangle into $K = l \times m$ regions of unit size, which means the above difference always vanishes. □

Note that depending on the number s and the distribution of anti-A-contaminated noise, a noise-concentrated block can be found without dividing the nation into regions of unit size. This fulfills the conclusion of the lemma.

The following theorems establish the relation between the size of the anti-A-noise-concentrated area and the total size of anti-A-noise-contaminated regions in the worst case, which in turn gives lower bounds to a breakdown point of decision, where a reversal of the decision takes place.

Theorem 2.1. *Let S_c be the size of the anti-A-noise-concentrated area and S_r be the total size of the anti-A-noise-contaminated regions of K-partitioned regional voting. We then have*:

(1) $S_r/S_c \leqslant (\lceil m_n/m_r \rceil + 1)^2 \times (m_r^2/m_n^2)$.
(2) $S_c < (m_n^2/m_r^2) \times 1/(\lceil m_n/m_r \rceil + 1)^2 \times 50\%N$ *is a sufficient condition for the regional voting to retain the original candidate selection of candidate A.*

Proof.

(1) For $m_r > m_n$, each anti-A-noise-concentrated block of size $m_n \times m_n$ can at most contaminate 4 $(= (\lceil m_n/m_r \rceil + 1)^2)$ regions. For $m_n \geqslant m_r$, we can also easily know that each anti-A-noise-concentrated block can at most contaminate $(\lceil m_n/m_r \rceil + 1)^2$ regions of size $m_r \times m_r$. The conclusion of item 1 follows immediately.

(2) First, when $S_c < (m_n^2/m_r^2) \times 1/(\lceil m_n/m_r \rceil + 1)^2 \times 50\%N$, we have $S_r \leqslant (\lceil m_n/m_r \rceil + 1)^2 \times 1(m_r^2/m_n^2) \times S_c < 50\%N$. That means that less than 50% of the K regions can be contaminated by noise. Thus, when $S_c < (m_n^2/m_r^2) \times 1/(\lceil m_n/m_r \rceil + 1)^2 \times 50\%N$, the regional voting will retain the original candidate selection of A. □

Because Assumption 2.3 implies that the original S_c has $S_c \times A\%$ votes for A, we have the following Corollary.

Corollary 2.1. *The original candidate selection of A can accommodate at least $(m_n^2/m_r^2) \times 1/(\lceil m_n/m_r \rceil + 1)^2 \times (A\%/2)N$ anti-A-noise (i.e., $(m_n^2/m_r^2) \times 1/(\lceil m_n/m_r \rceil + 1)^2 \times 50\%$ of the cells voting for A), before the candidate selection is reversed.*

Theorem 2.1 and Corollary 2.1 show clearly that to retain the original candidate selection of A in the regional voting, subdividing the nation into smaller sized regions leads to a higher stability margin, provided that Assumption 2.3 on the region size remains valid.

2.3. Further improvement by shifting strategy

The bounds of Theorem 2.1 and Corollary 2.1 can be further improved by exploiting the *Shifting Strategy* of [3]. We first define a *Shifting Strategy* for operation Λ with respect to a partitioned rectangular nation.

Shifting strategy for certain Action Λ

Consider the partitioning of a rectangular nation into $m_r \times m_r$ sized regions.

Repeat step (1) to step (2) m_r times:

(1) Move all the vertical partition lines to the right by one cell, then repeat step (2) m_r times;

(2) Move all the horizontal partition lines up by one cell, and execute Action Λ.

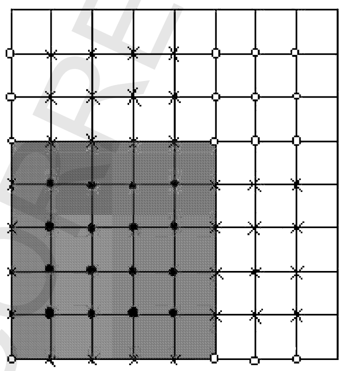
The shifting strategy enumerates all the possible different partitionings of the nation. Now by replacing Action Λ of the strategy with the regional voting subject to the same noise environment, we show by Theorem 2.2 below how we can improve Theorem 2.1.

Theorem 2.2. *Under the assumptions of Theorem* 2.1, *the shifting strategy ensures that there exists at least one partition satisfying the following properties*:

(1) $S_r/S_c \leqslant ((m_r + m_n - 1)/m_n)^2$.
(2) *The sufficient condition for the regional voting to retain the original candidate selection of A can be improved to* $S_c < (m_n/(m_r + m_n - 1))^2 \times 50\% N$.

Proof. To prove item (1), we first show that among all the possible m_r^2 different partitions that the Shifting Strategy can possibly generate, each of $m_n \times m_n$ size anti-A-noise-concentrated block is capable of contaminating the total of $(m_n + m_r - 1)^2$ different regions.

(1) For $m_r \geqslant m_n$, among all possible m_r^2 partitions, $(m_n - 1)^2$ partitions divide the block into 4 regions, $2 \times (m_n - 1) + 2 \times (m_n - 1)(m_r - m_n)$ partitions divide the block into 2 regions, while $(m_r - m_n + 1)^2$ partitions can not divide the block into more than one



Note: the gray part is a noise block.

Fig. 2. Different partitioning in the nation.

region. Summing them up, the noise-concentrated block is divided into $(m_n + m_r - 1)^2$ different regions. Fig. 2 illustrates the three different types of partitions showing how noise blocks are divided up into partitioned regions for the particular case of $m_n = 5$ and $m_r = 8$, for example. We have a total of 64 $(= 8^2)$ different partitions. By shifting the $m_r = 8$ size partitioned region, we see that a noise block is divided up into 4 different regions if one of the corners goes through any one of the solid dots, thus affecting or contaminating the neighboring 4 partitioned regions; similarly into two regions if the corner goes through one of the crossed points, but none if through any one of the hollow dots. Hence out of 64 partitions, 16 $(= (5-1)^2$ of such partitions affect 4 regions, 32 $(= 2\times(5-1) + 2\times(5-1)(8-5))$ affect 2 regions, while the remaining 16 $(= (8-5+1)^2$ can affect only a single regions.

(2) For $m_r < m_n$, consider all the possible different partitions and enumerate how many $m_r \times m_r$ joined regions can the block of size $m_n \times m_n$ be divided into. We enumerate each of all the $m_r \times m_r$ possible partitions, and we see that the $m_n \times m_n$ block would be divided into

$$\left\lceil\frac{m_n}{m_r}\right\rceil\cdot\left\lceil\frac{m_n}{m_r}\right\rceil, \left\lceil\frac{m_n}{m_r}\right\rceil\cdot\left(\left\lceil\frac{m_n-1}{m_r}\right\rceil+1\right), \left\lceil\frac{m_n}{m_r}\right\rceil\cdot\left(\left\lceil\frac{m_n-2}{m_r}\right\rceil+1\right),$$
$$\left\lceil\frac{m_n}{m_r}\right\rceil\cdot\left(\left\lceil\frac{m_n-3}{m_r}\right\rceil+1\right),\ldots, \left\lceil\frac{m_n}{m_r}\right\rceil\cdot\left(\left\lceil\frac{m_n-m_r+1}{m_r}\right\rceil+1\right);$$

$$\left(\left\lceil\frac{m_n-1}{m_r}\right\rceil+1\right)\cdot\left\lceil\frac{m_n}{m_r}\right\rceil, \left(\left\lceil\frac{m_n-1}{m_r}\right\rceil+1\right)\cdot\left(\left\lceil\frac{m_n-1}{m_r}\right\rceil+1\right),$$
$$\left(\left\lceil\frac{m_n-1}{m_r}\right\rceil+1\right)\cdot\left(\left\lceil\frac{m_n-2}{m_r}\right\rceil+1\right), \left(\left\lceil\frac{m_n-1}{m_r}\right\rceil+1\right)\cdot\left(\left\lceil\frac{m_n-3}{m_r}\right\rceil+1\right),\ldots,$$
$$\left(\left\lceil\frac{m_n-1}{m_r}\right\rceil+1\right)\cdot\left(\left\lceil\frac{m_n-m_r+1}{m_r}\right\rceil+1\right);$$

$$\left(\left\lceil\frac{m_n-2}{m_r}\right\rceil+1\right)\cdot\left\lceil\frac{m_n}{m_r}\right\rceil, \left(\left\lceil\frac{m_n-2}{m_r}\right\rceil+1\right)\cdot\left(\left\lceil\frac{m_n-1}{m_r}\right\rceil+1\right),$$
$$\left(\left\lceil\frac{m_n-2}{m_r}\right\rceil+1\right)\cdot\left(\left\lceil\frac{m_n-2}{m_r}\right\rceil+1\right), \left(\left\lceil\frac{m_n-2}{m_r}\right\rceil+1\right)\cdot\left(\left\lceil\frac{m_n-3}{m_r}\right\rceil+1\right),\ldots,$$
$$\left(\left\lceil\frac{m_n-2}{m_r}\right\rceil+1\right)\cdot\left(\left\lceil\frac{m_n-m_r+1}{m_r}\right\rceil+1\right);\ldots;$$

$$\left(\left\lceil\frac{m_n-m_r+1}{m_r}\right\rceil+1\right)\cdot\left\lceil\frac{m_n}{m_r}\right\rceil, \left(\left\lceil\frac{m_n-m_r+1}{m_r}\right\rceil+1\right)\cdot\left(\left\lceil\frac{m_n-1}{m_r}\right\rceil+1\right),$$
$$\left(\left\lceil\frac{m_n-m_r+1}{m_r}\right\rceil+1\right)\cdot\left(\left\lceil\frac{m_n-2}{m_r}\right\rceil+1\right),$$
$$\left(\left\lceil\frac{m_n-m_r+1}{m_r}\right\rceil+1\right)\cdot\left(\left\lceil\frac{m_n-3}{m_r}\right\rceil+1\right),\ldots,$$
$$\left(\left\lceil\frac{m_n-m_r+1}{m_r}\right\rceil+1\right)\cdot\left(\left\lceil\frac{m_n-m_r+1}{m_r}\right\rceil+1\right)$$

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regions separately. Summing up all the possible terms above, we arrive at the following formula:

$$\sum_{i=0}^{m_r-1} \left\lceil \frac{m_n - i}{m_r} \right\rceil = m_n.$$

Now we have proved that an $m_n \times m_n$ block will be divided into $(m_n - m_r + 1)^2$ different regions for all the $m_r \times m_r$ different partitions.

According to the Pigeon Hole Principle [1], there should exist at least one partition in which the existing S_c/m_n^2 (= the total size of anti-A-noise-concentrated area as expressed by equivalent number of noise concentrated blocks) anti-A-noise-concentrated blocks contaminate at most $(S_c/m_n^2)(m_n - m_r + 1)^2/m_r^2$ regions. For this special partition,

$$S_r \leqslant (S_c/m_n^2)(m_n - m_r + 1)^2/m_r^2 \times m_r^2 = S_c\big((m_n - m_r + 1)/m_n^2\big)^2.$$

The conclusion of item (1) now follows immediately.

Item (2) follows from item (1) since a sufficient condition for retaining the original candidate selection of A requires that the number of noise-contaminated regions is to be less than 50% of the total number of regions. □

Corollary 2.2. *If a noise-contaminated region and a partition size are fixed, we can find at least one partition such that a specific partition can accommodate at least*: $(m_n/(m_r + m_n - 1))^2 \times (A\%/2)N$ *anti-A-noise-contaminated votes* (*that is* $(m_n/(m_r + m_n - 1))^2 \times 50\%$ *of all the votes originally given to* A) *before the original result of candidate selection is reversed.*

Conjecture. *Theorems* 2.1 *and* 2.2 *and related corollaries remain valid for a more general* G, *including features matching by PCA analysis.*

We will verify this conjecture by an experiment in Appendix A.

3. Conclusion and discussion

The detailed analysis of regional and national voting shows that regional voting is the more stable and robust of the two. This is in agreement with our physical intuition as supported by Lemma 2.1, that only a small fraction of all the regions absorb the heavily concentrated effects of noise. We would like to give some concrete examples numerically below.

3.1. Conclusion

3.1.1. Robustness of regional voting

Table 1 is computed from the formula of Corollary 2.1 and Observation 2.1 of Section 2.2 for several values of $(A\% - B\%)$.

Table 1
Stability margins of regional voting and national voting for $N = 10000$

$A\% - B\%$	Regional voting		National voting
	$m_n/m_r = 1$	$m_n/m_r = 2$	
5%	656	1167	250
10%	688	1222	500
20%	750	1333	1000

Stability margins of anti-A-noise-contaminated votes which regional and national voting can accommodate before the decision reverses.

Table 2
Improved regional voting by shifting strategy calculated for $N = 10000$

$A\% - B\%$	$m_n = 3,\ m_r = 3$		$m_n = 4,\ m_r = 2$	
	Corollary 2.1	Corollary 2.2	Corollary 2.1	Corollary 2.2
5%	656	945	1167	1680
10%	688	990	1222	1760
15%	719	1035	1278	1840
20%	750	1080	1333	1920

We see that the regional voting is always more stable and robust than the national voting when $A\% - B\%$ is not too large, say 5% and 10%, and this robustness increases as m_n/m_r increases or the partitioned region size becomes smaller.

For larger values of $A\% - B\%$, say, 20%, at $m_n/m_r = 1$, the lower bound of regional voting given by Corollary 2.1 is smaller than the exact bound of national voting. We still believe that the regional voting will be more stable than the national voting, because in counting the number of possible "losing" regions in Theorem 2.1, we have counted the number of all the noise-contaminated regions, including those which still have a margin to the breakdown point retaining the pro-A region. In fact, we have excluded only those regions entirely *clean or free of any noise*.

3.1.2. Improvement by shifting strategy

Given some distributed noise-concentrated area as the union of all noise-concentrated blocks, Theorem 2.2 and Corollary 2.2 provide a method of improving the stability of regional, matching as shown in Table 2. A larger improvement is evident for smaller $A\% - B\%$.

3.1.3. A tradeoff on region size

By examining the theorems, we see that there must be a tradeoff for size m_r of the partitioned regions; reducing the size of the partitioned region increases the robustness of regional voting, but Assumption 2.3 requires the size to be not too small. This is remarkably well born out experimentally in Fig. 5 where both at low (25%) as well as at high (50%) noise levels, the recognition rate falls off if the entire image of 80×120 pixels is divided into more than 384 regions with the region size corresponding to 5×5 pixels. This is so because the original distribution of undisturbed area can not be expected to hold. The reader should also note that when the number of partitioning is small, including the national voting of one partitioning, the recognition rate also falls off, implying that the global matching is not efficient.

3.2. Discussion

3.2.1. What if there are more than two candidates?

If the number of candidates exceeds two, the number of decision-making processes increases. For example, we may allow each region to select the top two or more candidates at a time, then make the candidate selection based on the summed results of all the regions. Here, we set up one simple model where the basic decision-making principle adopted in the two-candidate system is retained. Each of the regions selects only one (1) candidate according to a simple majority principle, and then the regional voting selects one candidate who wins a majority of the winning regions. Suppose there are candidates $A, B, C, \dots,$ and $A\% > B\% > C\% > \cdots$. The anti-A-noise is defined to convert the votes originally for A to B and keep other votes unchanged. We have the following theorem by exactly the same proof of Theorem 2.1:

Theorem 3.1.

(1) *National voting can accommodate at most only* $(A\% - B\%)/2 \cdot N$ *anti-A-noise-contaminated-votes, i.e.,* $(A\% - B\%)/2A\%$ *among all the votes originally for* A.
(2) *Regional voting can accommodate at least* $(m_n^2/m_r^2) \times 1/(\lceil m_n/m_r \rceil + 1)^2 \times (A\%/2)N$ *anti-A-noise-contaminated votes, i.e.,* $(m_n^2/m_r^2) \times 1/(\lceil m_n/m_r \rceil + 1)^2 \times 50\%$ *among all the votes originally for* A.

We have entirely the same conclusion as in the two-candidate system, confirming that the regional voting still accommodates a higher level of noise when $A\%$ and $B\%$ are very close.

3.2.2. Effect of salt-and-pepper noise

In sharp contrast to the localized, and thus concentrated, noise we have assumed in the present paper, we will now examine the effects of impulse-type salt-and-pepper noise [6].

Definition 3.1. White noise is a set of noise "uniformly" distributed over the nation in such a way that in each of all reasonably large sized areas whether composed of a continuous part of the nation or of randomly chosen blocks of cells, we have the same percentage of noise.

Consider only impulse-type well dispersed anti-A-noise. A similar conclusion about the noise bound as Observation 2.1 can be obtained:

Observation 3.1. *Global voting and regional voting can accommodate the same percentage of salt-and-pepper noise.*

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The observation could easily be proved by noting that in each region or the whole nation, the candidate selection will not reverse unless there is less than $\frac{1}{2}(A - B)\%$ of salt-and-pepper noise.

The observation shows that as long as the partitioning region is large enough to allow the original distribution of the entire nation to prevail, we expect no difference between the two decision systems in the presence of salt-and-pepper noise. It is when the uniform distribution assumption between national voting and regional voting fails that the difference matters.

3.2.3. Size of noise-concentrated blocks

The generality of our analysis depends heavily on the fact that a possible error between the total size of all noise-affected cells and the union of all non-overlapping noise-concentrated blocks is kept negligibly small. Let us cut some $m_n \times m_n$ blocks out of a reasonably concentrated noise-contaminated area. It is reasonable to assume that $m_n \times m_n$ is small compared to the size of any continuous parts of noise-contaminated area. We expect that a measure of what is left after cutting out blocks is quite small compared with the total amount of noise. We demonstrate this formally below by showing why our theorems and corollaries should remain valid.

Definition 3.2. A line segment is called an orthodiameter of a continuous area of a nation, *if and only if*

(1) all the points of the segment lie within the area;
(2) only the two end points of the line segment lie on the boundary of the area;
(3) the line segment is parallel to the horizontal lines or the vertical lines comprising the boundary of the nation.

Definition 3.3. The orthomeasure of an area (continuous or detached) of a nation is defined by the length of the shortest orthodiameter of any continuous part of the area.

Observation 3.2. *Let us cut out as many $m_n \times m_n$ blocks as possible and let S_c be the total size of all these blocks* (*i.e., the size of the noise-concentrated area discussed in Definition* 2.2). *Let* OM *be the orthomeasure of the set of noise influenced votes of the nation, and N_n be the number of noise* (*or, the number of noise-affected cells*). *We have*:

$$\lim_{\frac{OM}{m_n - 1} \to \infty} \frac{N_n - S_c}{N_n} = \lim_{\frac{OM}{m_n - 1} \to \infty} \frac{N_n - S_c}{S_c} = 0.$$

Note that the above observation includes the situation that $N_n - S_c = 0$ when $m_n = 1$.

3.2.4. What if only a fraction of the anti-A-noise-concentrated blocks are effective?

Suppose that in a noise-concentrated area, only $r\%$ of votes for A undergo changes to B where r is some constant within [0, 1], possibly close to 1. All the results in Theorems 2.1–2.2 remain the same, while those of Corollaries 2.1–2.2 may be divided by r. We believe that the conclusion of Section 3.1 still holds, because in all the corollaries we always use much larger values in estimating lower bounds, as we have thrown away all the regions contaminated by even a single unit of noise.

3.2.5. Relaxing the average distribution assumption

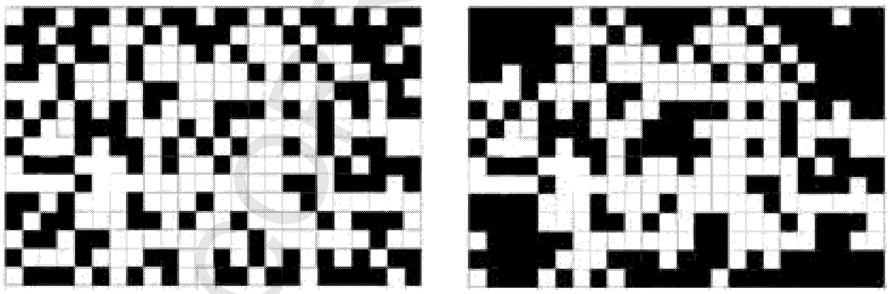
In view of the proofs of Theorems 2.1 and 2.2 given in Section 2, the restriction on regional size in Assumption 2.3 in Section 2.2 is too strict and can be relaxed as follows:

Observation 3.3. *All the conclusions of Theorems* 2.1 *and* 2.2 *and Corollaries* 2.1 *and* 2.2 *still hold, as long as we can choose the size of partitioning regions large enough such that the voting distributions in the absence of noise for Candidates A and B satisfy* $A\% > B\%$ *in each of the regions.*

We require only that in the absence of noise the integrated effects within each of the regions satisfy the relation $A\% > B\%$, and we emphasize that the voting distributions need not follow that of the nation.

4. Experiment: White or black-dominated flags

The first example relates to a white-black mixed flag which we want to recognize either as a white or a black-dominated flag (see Fig. 3 for illustration, where the cells in the figure denote a smallest unit of a "pixel"). Unlike the second example to follow, this example applies the present theory directly on a pixel by pixel basis without resorting to features extraction or data compression methods using a transformation such as the Turk & Pentland method. The size of a "nation" chosen is 360 cells ($= 15 \times 24$), and a partitioned region consists of either 4×5 cells or 3×3 cells, which we believe to be neither too large nor too small relative to the size of the nation. We now generate a white-dominated flag of Fig. 3(a) randomly as follows. We set up a short and simple program running over each of 360 cells such that each cell has a probability of 0.4 to be black and 0.6 to be white. A typical white dominated flag generated is shown in Fig. 3(a), where there are 207 white cells and 153 black cells. In global voting, "White" gets 207 votes while "Black" 153 votes; by regional vote counting based on 4×5 regional partitioning, "White" wins in 12 regions, while



(a) (b)

Fig. 3. "White" or "Black" dominated?

"Black" does in 4 regions, and within another 2 regions, "White" and "Black" get the same votes. If we further divide the nation into 3×3 sized regions, "White" wins in 28 regions, while "Black" does in 12 regions in regional voting. Now we randomly choose 7 pixels of Fig. 3(a) and introduce anti-White-noise blocks of a 5×5 region so that every "white" pixel within the block is transformed to a Black pixel with a probability of "0.7". As a result, 35 "White" pixels are changed to "Black" by anti-white-noise contaminated blocks transforming Fig. 3(a) to Fig. 3(b). By counting, we see that after the noise is added, the global voting will reverse the results of the candidate selection from "White"- to "Black"-dominated because this time "Black" gets 188 votes, while "White" gets only 172 votes in global voting. But, by the regional voting having the regions of size 4×5, the original selection of "White"-dominated still remains valid, because this time "White" wins in 10 regions while "Black" does so in 6, and within another 2 regions, "White" and "Black" get the same votes. If we further divide the nation into 3×3 sized regions, we will see that "White" wins in 25 regions, while "Black" does in 15 regions in regional voting, increasing the stability margin, thus supporting our theory.

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Appendix A. Facial recognition

Among many dimension-reducing algorithms developed for facial recognition, Turk and Pentland's eigenvector algorithm [7] has been proved most effective. Using images of 16 people in Turk and Pentland's original work, we show a most convincing verification of the conjecture in practical image processing applications by carrying out a set of facial recognition experiments subjecting the images to artificially produced, locally concentrated noise in conformity with our theory. The first set of images are "free of noise" as shown in Fig. 4(a). In Figs. 4(b) and (c), we introduce circular blocks of noise of Photoshop's version 4.0 randomly into the independent test images (but of the same 16 people) at low and high noise levels of 25% and 50% levels, respectively(see Fig. 4).

Circular blocks of noise introduced by Photoshop have a density variation within the area ranging from levels 0 to 255. We have defined the area to be noise-affected if the density levels of the original image and those of the noise-affected image differ by 64 (i.e., $25\% \times 256$) in pixel density. The noise level in each of the images is shown in Table 3. Each picture is of size $80 \times 120 = 9600$ pixels. The first row of each level indicates the numbers of noise, the second row the percentage of the noise.

Turk and Pentland's eigenvector algorithm [7] is chosen as the features extraction or data compression method, where the eigenvector transformation operates uniformly not only over each of the discrete pixels in the nation, but also over each of the pixels of the

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(a) (b) (c)

Fig. 4. Typical noise contaminated images at low and high noise level. (a) Original image. (b) Lower noise level. (c) Higher noise level.

Table 3
The noise of each images

Noise suffered face images		Training face images of individuals (80 × 120 size)															
		No. 0	No. 1	No. 2	No. 3	No. 4	No. 5	No. 6	No. 7	No. 8	No. 9	No. 10	No. 11	No. 12	No. 13	No. 14	No. 15
Lower level	No.	2757	2450	2429	1871	2004	1729	1959	2743	2502	2379	2043	2437	1566	2895	2517	1946
	%	28.7	25.5	25.3	19.5	20.9	18.0	20.4	28.6	26.1	24.8	21.3	25.4	16.3	3 0.2	26.2	20.3
Higher lever	No.	5748	5474	5701	4372	4909	5051	5359	4464	4907	5073	4755	4900	4762	5377	5711	4811
	%	59.9	57.0	59.4	45.5	51.1	52.6	55.8	46.5	51.1	52.8	49.5	51.0	4 9.6	56.0	59.5	50.1

The first row of each level indicates the numbers of noise, while the second gives the level of noise in percentage.

Table 4
National voting results at lower noise level

New face images	Training face images of individuals															
	No. 0	No. 1	No. 2	No. 3	No. 4	No. 5	No. 6	No. 7	No. 8	No. 9	No. 10	No. 11	No. 12	No. 13	No. 14	No. 15
No. 0	18.93	20.23	34.60	24.04	30.77	28.02	40.80	31.51	23.85	25.44	13.48	22.39	14.01	20.37	27.54	26.90
No. 1	25.55	13.15	32.00	25.41	30.37	21.44	32.73	22.29	28.07	25.60	12.67	25.63	5.99	21.39	21.93	27.79
No. 2	14.76	13.79	16.63	15.20	25.21	19.23	38.04	21.97	17.95	17.06	10.74	17.09	11.98	14.20	20.82	18.16
No. 3	24.17	15.79	24.87	**9.3**	26.67	20.58	34.55	21.74	23.18	20.93	13.36	23.30	13.11	15.25	20.83	16.56
No. 4	15.50	13.79	20.19	13.87	10.50	22.63	19.31	21.62	17.15	13.10	10.28	18.38	17.59	9.89	17.25	10.21
No. 5	26.01	16.43	30.25	22.41	31.77	8.37	32.78	18.79	27.93	27.93	13.66	22.33	5.25	21.17	20.82	23.57
No. 6	36.30	25.85	38.31	30.69	28.12	30.79	**11.03**	22.58	36.84	28.94	20.60	30.74	17.70	23.34	29.48	26.42
No. 7	57.05	47.52	64.74	51.66	61.36	45.01	61.24	**18.46**	58.02	59.58	42.23	53.26	20.90	44.63	54.94	33.06
No. 8	27.50	21.36	34.06	22.87	31.65	29.53	36.25	26.57	21.06	26.41	16.10	30.42	12.95	22.22	27.31	19.84
No. 9	23.82	15.54	31.05	22.60	28.35	22.84	33.05	27.12	21.55	16.41	11.86	22.59	11.51	16.08	22.06	27.06
No. 10	28.49	21.65	38.67	28.74	39.99	30.03	46.45	33.03	28.99	30.58	15.4 3	31.23	10.07	26.74	30.41	31.58
No. 11	20.94	14.24	22.01	12.80	16.10	18.67	26.01	17.08	21.05	17.26	11.2 4	15.16	13.40	9.30	18.68	6.99
No. 12	50.44	35.56	53.11	42.45	53.29	31.18	48.50	23.75	50.48	50.75	31.5 9	48.17	**8.27**	39.89	42.54	34.18
No. 13	33.37	27.18	39.58	28.87	34.45	35.50	40.61	23.85	31.81	30.14	22.8 4	32.23	16.36	18.87	34.66	20.64
No. 14	29.65	18.59	35.83	21.68	25.63	28.30	30.12	25.94	31.97	24.90	14.1 3	26.25	13.41	19.35	19.57	17.93
No. 15	44.76	40.83	50.45	35.25	42.80	48.21	55.28	35.42	44.01	42.58	34.7 4	47.35	30.41	32.66	46.81	**11.23**

partitioned regions. Because the effects of random noise remain random on the transformed planes without being magnified or filtered, we assume that the effects of noise blocks remain transparent to the transformation applied, implying that the conjecture of Section 2 remains valid. For convenience, we now resort to a new partition notation dividing the

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Table 5
16 regional voting at the lower noise level

New face images	Training face images of individuals															
	No. 0	No. 1	No. 2	No. 3	No. 4	No. 5	No. 6	No. 7	No. 8	No. 9	No. 10	No. 11	No. 12	No. 13	No. 14	No. 15
No. 0	3	0	0	0	0	0	0	2	0	0	4	0	6	0	0	1
No. 1	0	2	0	1	1	1	0	4	0	0	3	0	2	0	1	1
No. 2	1	0	2	1	0	0	0	2	0	2	1	0	5	1	0	1
No. 3	0	1	0	**5**	0	0	0	2	0	0	2	0	3	0	0	3
No. 4	0	0	0	0	**6**	0	0	1	2	1	2	0	1	2	0	1
No. 5	0	0	0	1	0	**5**	0	2	0	0	1	0	4	1	0	2
No. 6	0	0	0	0	0	0	**5**	3	0	1	3	0	1	1	1	1
No. 7	0	0	0	0	0	0	0	**8**	0	0	1	1	5	0	0	1
No. 8	0	1	0	0	1	0	0	3	**5**	0	2	0	2	1	0	1
No. 9	0	0	0	0	0	0	0	0	0	**6**	1	0	4	1	0	4
No. 10	0	0	0	0	0	0	0	1	0	0	7	0	8	0	0	0
No. 11	0	1	0	0	0	0	0	1	1	0	3	4	2	0	0	4
No. 12	0	0	0	0	0	0	0	0	0	1	0	0	**13**	0	0	2
No. 13	0	0	0	0	0	1	0	5	0	0	3	0	2	4	0	1
No. 14	0	0	0	2	0	1	0	1	0	0	1	0	2	3	**4**	2
No. 15	0	0	0	0	0	0	0	1	0	0	0	0	4	1	0	**10**

Table 6
384 regional voting at lower noise level

New face images	Training face images of individuals															
	No. 0	No. 1	No. 2	No. 3	No. 4	No. 5	No. 6	No. 7	No. 8	No. 9	No. 10	No. 11	No. 12	No. 13	No. 14	No. 15
No. 0	**110**	3	1	20	4	5	1	27	13	2	11	8	43	1	2	15
No. 1	60	**83**	4	4	4	12	4	22	7	1	13	8	31	3	4	6
No. 2	61	14	**66**	2	2	12	2	19	8	3	6	11	47	4	1	8
No. 3	13	22	4	**121**	0	15	3	17	13	3	4	1	28	4	1	17
No. 4	44	15	6	6	**122**	1	2	12	5	4	3	4	6	6	11	19
No. 5	30	9	2	5	4	**117**	4	18	11	2	4	2	36	3	2	17
No. 6	55	9	6	9	10	4	**95**	25	5	7	8	5	8	7	5	8
No. 7	27	2	3	29	5	5	1	**123**	11	0	2	0	38	0	2	18
No. 8	43	13	4	3	13	14	3	14	**96**	0	4	3	25	8	2	21
No. 9	63	16	1	2	5	7	3	13	10	**85**	11	4	23	7	5	11
No. 10	53	20	5	4	0	14	6	15	17	2	**60**	4	47	2	3	14
No. 11	40	4	2	7	9	6	3	34	12	1	7	**86**	29	5	3	18
No. 12	25	0	3	2	6	0	3	17	9	1	5	2	**166**	5	2	24
No. 13	21	9	4	8	6	17	3	34	15	3	7	7	39	**61**	8	24
No. 14	28	13	5	7	17	5	1	2	17	2	5	12	6	33	**98**	19
No. 15	3	5	6	2	4	4	2	16	10	4	3	7	36	2	3	**159**

nation into 1 (namely global), 8, 16, 32, 64, ..., regions so that division into 1 region corresponds now to the original national matching.

Recognition rates of the experiments are compared in Tables 4–11. Tables 4 and 8 illustrate the results of the national voting, while Tables 5–6 and 9–11 those of the regional voting. In Tables 4 and 8, the numbers in each cell of the tables indicate the distances between test face images for identification and training face images obtained by the eigenface algorithm. The cells with a shortest distance along the row indicate that the test faces of corresponding rows are recognized as the training face image of the corresponding columns. The underlined cells indicate that the test faces are correctly recognized. In Tables 5–6 and 9–11, the numbers in each cell indicate the number of regions having the most votes from among the training face images as computed by the distances by eigenface algorithm. The underlined cells having the most votes along the

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Table 7
1600 regional voting at lower noise level

New face images	Training face images of individuals															
	No. 0	No. 1	No. 2	No. 3	No. 4	No. 5	No. 6	No. 7	No. 8	No. 9	No. 10	No. 11	No. 12	No. 13	No. 14	No. 15
No. 0	**353**	33	17	31	38	83	23	106	63	26	54	39	197	24	22	97
No. 1	173	**306**	30	19	17	69	28	126	47	15	67	32	177	26	27	47
No. 2	195	69	**252**	25	25	47	26	89	43	24	37	42	193	39	21	79
No. 3	78	64	39	**390**	11	51	28	75	59	29	37	25	165	40	17	78
No. 4	156	59	38	34	**497**	29	22	65	25	29	29	27	58	25	59	54
No. 5	140	49	35	15	23	**364**	37	111	49	7	21	15	217	25	15	83
No. 6	166	48	38	17	42	34	**386**	135	23	31	27	35	102	29	29	64
No. 7	110	29	28	22	18	46	20	**535**	44	6	20	9	218	14	10	77
No. 8	126	63	24	22	40	45	33	107	**338**	13	33	21	197	38	19	87
No. 9	181	68	25	31	27	43	45	81	42	**281**	38	29	153	41	36	85
No. 10	146	90	34	23	13	65	51	94	68	23	221	16	245	23	28	66
No. 11	96	38	19	19	33	79	16	96	43	17	40	**410**	135	38	23	104
No. 12	104	10	19	10	12	8	19	103	35	10	17	15	**723**	18	9	94
No. 13	92	59	21	34	33	91	41	163	51	18	42	26	181	**241**	25	88
No. 14	75	60	19	58	35	79	26	103	15	31	47	39	126	38	**342**	113
No. 15	26	34	28	9	23	54	28	106	40	18	31	38	185	22	19	**545**

Table 8
National voting results at higher noise level

New face images	Training face images of individuals															
	No. 0	No. 1	No. 2	No. 3	No. 4	No. 5	No. 6	No. 7	No. 8	No. 9	No. 10	No. 11	No. 12	No. 13	No. 14	No. 15
No. 0	51.06	41.67	65.32	46.40	58.01	49.70	63.53	40.14	51.47	51.90	33.14	47.90	20.60	40.60	51.56	36.38
No. 1	66.30	50.64	77.58	60.89	71.85	57.65	71.93	41.34	68.32	67.05	44.73	64.76	22.13	52.91	64.00	48.24
No. 2	65.98	50.77	71.64	61.63	72.51	55.34	73.81	36.84	66.15	64.90	42.89	62.86	20.98	52.87	62.41	46.42
No. 3	56.15	43.63	60.18	41.97	56.69	46.08	54.90	28.73	55.42	52.75	34.37	51.68	17.24	40.93	51.45	33.54
No. 4	51.35	43.09	63.12	47.23	49.70	55.13	49.44	41.26	50.91	50.45	36.45	55.07	24.59	40.12	51.36	29.02
No. 5	65.16	54.20	76.91	61.63	72.55	51.03	71.83	36.53	65.58	68.71	44.70	63.59	22.56	54.23	65.56	41.06
No. 6	63.76	50.91	73.58	56.73	63.87	56.23	52.38	37.00	64.46	60.65	44.85	60.27	23.86	47.51	61.10	36.13
No. 7	85.92	69.10	91.66	75.78	88.26	67.38	86.01	37.45	83.34	86.50	61.70	80.78	33.07	66.09	80.95	49.05
No. 8	74.93	58.73	85.98	65.82	78.11	64.27	73.37	46.40	69.80	71.08	49.17	69.64	25.16	59.61	69.13	52.86
No. 9	50.94	37.82	61.12	42.17	53.94	48.70	60.87	41.90	48.88	45.76	31.33	48.52	21.02	36.11	49.31	35.05
No. 10	73.39	58.18	87.89	67.41	81.47	66.05	77.33	48.36	73.89	74.41	49.3 4	70.65	27.07	60.92	70.07	53.96
No. 11	50.85	39.86	58.21	42.23	51.38	48.06	53.82	30.05	49.98	48.11	33.9 7	45.73	18.91	36.76	49.25	28.44
No. 12	81.31	67.12	90.68	72.58	86.60	65.15	80.58	44.14	81.97	81.68	57.3 4	78.33	**28.76**	66.69	76.86	48.72
No. 13	75.64	61.30	85.88	67.11	75.95	66.50	71.06	43.20	75.97	72.87	51.2 1	70.01	29.22	55.62	71.56	46.47
No. 14	75.89	58.37	88.50	66.87	79.84	64.20	73.15	49.93	74.52	72.58	51.6 7	71.31	25.46	60.73	67.14	51.30
No. 15	99.74	82.16	111.65	86.29	98.43	84.81	97.85	59.14	98.75	98.41	73. 36	93.96	44.00	79.34	92.03	54.42

corresponding rows indicate that the test faces of the rows are correctly recognized by the voting principle. Note that the total numbers of each row in Tables 6, 7, 10 and 11 are much less than the numbers of partitioned regions. This is because the determinants of image matrices of some regions vanish and are singular, so that no eigenvalues exist. This happens if many of the pixel values are the same as in the background image.

The superiority of the regional voting over the national voting is evident for images of lower noise level. For 16 partitioning, 10 images out of 16 candidates are recognized correctly (Table 5) while for 384 regional partitioning, all 16 images are recognized correctly (Table 6). The Turk and Pendland's [7] national voting method can recognize only 5 images out of 16 (Table 4).

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Table 9
16 regional voting at higher noise level

New face images	Training face images of individuals															
	No. 0	No. 1	No. 2	No. 3	No. 4	No. 5	No. 6	No. 7	No. 8	No. 9	No. 10	No. 11	No. 12	No. 13	No. 14	No. 15
No. 0	0	0	0	0	0	0	0	1	0	1	1	0	9	1	0	3
No. 1	0	0	0	0	1	1	0	2	0	0	1	0	7	0	0	4
No. 2	0	0	1	0	0	0	0	2	0	1	0	0	7	0	0	5
No. 3	0	0	0	1	1	0	0	3	0	0	2	0	6	0	0	3
No. 4	0	0	0	0	1	0	0	2	0	0	2	1	6	1	0	3
No. 5	0	0	0	0	1	0	0	3	0	0	0	0	7	0	0	5
No. 6	0	0	0	1	0	0	0	6	0	0	1	0	3	0	0	5
No. 7	0	0	0	0	0	0	0	3	0	0	0	0	9	0	0	4
No. 8	0	0	0	0	1	0	0	2	1	1	0	0	7	0	0	4
No. 9	0	0	0	1	0	0	0	3	0	1	1	1	6	0	0	3
No. 10	0	0	0	0	1	0	0	1	0	0	1	0	9	1	0	3
No. 11	0	0	0	0	0	0	0	4	0	0	3	1	3	0	0	5
No. 12	0	0	0	0	0	0	0	1	0	0	0	0	**8**	0	0	7
No. 13	0	0	0	0	0	0	0	4	0	0	0	0	7	1	0	4
No. 14	0	0	1	0	0	0	0	1	0	0	1	0	7	2	2	2
No. 15	0	0	0	0	0	0	0	1	0	0	1	0	8	0	0	6

Table 10
384 regional voting at higher noise level

New face images	Training face images of individuals															
	No. 0	No. 1	No. 2	No. 3	No. 4	No. 5	No. 6	No. 7	No. 8	No. 9	No. 10	No. 11	No. 12	No. 13	No. 14	No. 15
No. 0	**52**	3	3	12	5	10	3	37	24	2	10	5	51	7	4	38
No. 1	39	23	9	3	6	29	6	35	14	1	7	9	56	7	3	19
No. 2	51	3	26	3	4	10	5	37	16	0	11	4	58	3	3	32
No. 3	14	9	6	**64**	10	9	8	32	20	0	12	7	44	5	1	25
No. 4	18	15	4	13	**39**	14	3	29	13	3	13	6	31	4	18	43
No. 5	9	5	2	10	6	60	5	36	20	1	2	3	61	2	3	41
No. 6	22	2	7	6	13	17	32	43	12	5	7	6	47	5	3	39
No. 7	18	4	4	9	3	16	1	**88**	12	0	4	3	66	1	4	33
No. 8	26	2	3	3	6	9	5	24	58	3	8	4	64	6	0	45
No. 9	41	9	2	13	6	11	4	27	14	41	12	8	43	11	4	20
No. 10	26	14	8	1	6	2	11	40	20	2	27	3	68	7	3	28
No. 11	23	7	4	16	8	13	3	34	15	1	13	33	53	5	5	33
No. 12	7	0	4	20	4	14	5	39	10	0	6	4	**101**	4	2	46
No. 13	10	4	6	15	8	15	4	52	16	1	8	4	54	25	6	38
No. 14	20	9	5	30	6	5	6	23	13	0	11	7	56	4	40	31
No. 15	6	7	6	3	3	10	2	28	12	0	7	5	51	3	3	**120**

At a higher noise level, 16 partitioning recognizes only 1 image (Table 9), while 384 regional partitioning recognizes 6 (Table 10). This should be compared with the national voting of one correct recognition (Table 8).

Increasing the number of regions does not necessarily improve the result further. The numerical results for partitioning into 1600 regions, which are 15 and 2 out of 16 images for lower and higher noise level images, respectively, confirm this fact (see Tables 7 and 11, or Fig. 5). This is the trade-off problem on the size of the partitioning size as discussed in Section 3.1.3, and the this is most clearly demonstrated in Fig. 5.

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Table 11
1600 regional voting at higher noise level

New face images	Training face images of individuals															
	No. 0	No. 1	No. 2	No. 3	No. 4	No. 5	No. 6	No. 7	No. 8	No. 9	No. 10	No. 11	No. 12	No. 13	No. 14	No. 15
No. 0	164	31	24	28	29	92	35	196	80	15	44	33	278	33	22	102
No. 1	132	83	34	21	20	64	38	203	65	18	46	31	305	40	25	81
No. 2	167	34	98	16	19	48	37	193	60	9	34	22	300	32	19	118
No. 3	75	55	27	152	31	63	54	159	64	16	52	39	249	46	26	98
No. 4	82	53	34	35	135	92	37	142	48	38	47	44	206	33	66	114
No. 5	55	31	37	16	18	142	34	193	67	15	26	15	377	28	14	138
No. 6	74	30	35	29	48	57	136	236	45	26	21	25	245	42	24	133
No. 7	39	35	28	18	11	65	23	366	51	10	19	10	367	24	12	128
No. 8	107	34	22	17	30	55	36	175	187	14	35	21	323	29	12	109
No. 9	159	41	22	33	34	30	41	137	55	146	49	47	233	49	28	102
No. 10	79	53	44	20	26	75	52	187	68	14	68	18	333	40	11	118
No. 11	66	45	25	27	31	82	29	190	63	17	45	127	243	45	26	145
No. 12	55	19	20	30	15	46	34	203	49	9	21	16	**507**	30	14	138
No. 13	50	37	31	41	22	95	38	235	68	15	37	27	284	93	15	118
No. 14	66	34	25	54	30	45	32	157	49	22	24	31	310	36	145	146
No. 15	27	39	37	9	13	57	28	177	46	15	27	18	320	32	16	**345**

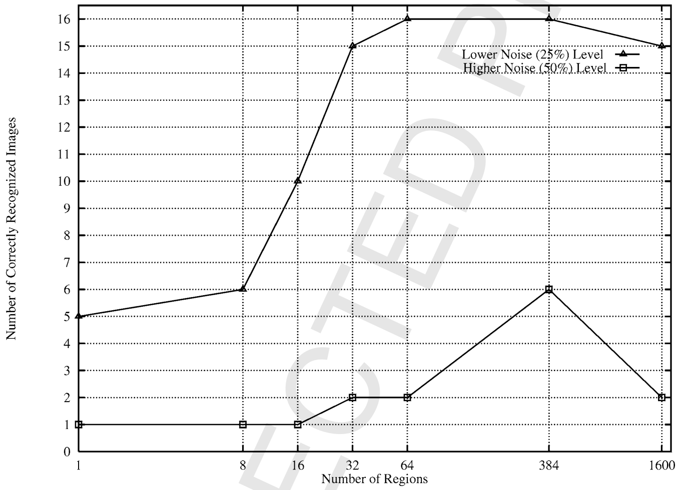

Fig. 5. Recognition rates of each regional matching scheme and national matching.

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