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A faster data assignment algorithm for maximum likelihood-based multitarget motion tracking with bearings-only measurements

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Abstract

We have proved a new rotational sorting algorithm capable of reducing the complexity of data assignment process embedded in the maximum likelihood (ML)-based solution of a multitarget tracking problem from $O(N^3)$ of the conventional Hungarian type routines to $O(N^2)$ provided that the bearings-only measurements from an array of passive sensors are free from cluttering and missing data. © 2001 Elsevier Science B.V. All rights reserved.

Keywords: Bearings-only measurement; Multitarget motion tracking; Data association problem; Hungarian algorithm

1. Introduction

We consider tracking the N targets having arbitrary motion exploiting the given k scans of bearings-only measurements from s sensors monitoring the given surveillance region. Now each "sensor" consists of a passive array of acoustic sensors and a front-end direction-of-arrival (DOA) estimator, and each scan of the sensors provides a set of measurements of N moving targets within the region. Fig. 1 illustrates a most typical multitarget–multisensor encounter. The purpose is to identify the targets at each of time sequences, estimating the targets' states as to their positions. We restrict the analysis to the case of no cluttering and no missing measurements throughout this paper.

Having an important application in implementing multitarget trackings in a spatial sensor tracker including an underwater passive sonar tracking environment, the problem has been extensively studied [1–6]. Despite extended efforts in the past, the problem has not been solved successfully as yet mostly because of the difficulty associated with the data association problem mainly because it remains NP hard when the number of the sensors exceed three even in the simplified cases with stationary targets [4,7,8].

The maximum likelihood (ML)-based relaxation approach has been one of the most extensively used methods in the problem, where a non-convex conditional likelihood function of bearings-only measurements are maximized with respect to both data associations computed and target initial states. This is

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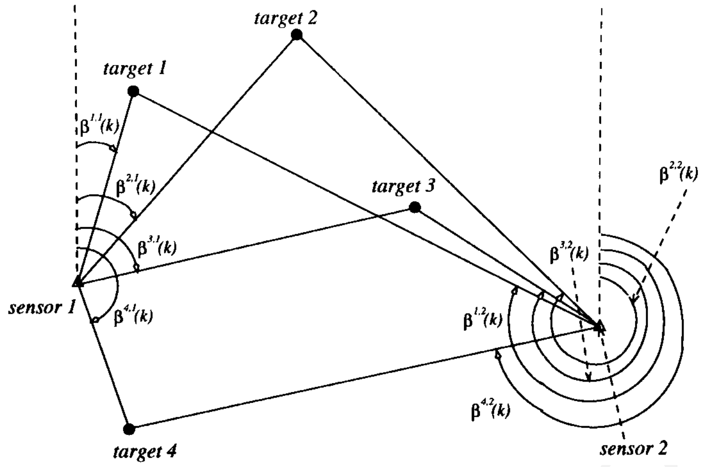


Fig. 1. Typical target–sensor encounter.

equivalent to minimizing the magnitude of the average square errors (ASE) of its exponent (see Eq. (5) in Section 3). A global minimum can be attained in principle by repeating the following two steps until the solution converges; the first step minimizing the ASE with respect to initial target state vector for a fixed data assignment matrices and the second step minimizing the ASE with respect to data assignment matrices constructed for the given initial state vector. The first step of the approach usually exploits the Gauss–Newton downhill type algorithm where resolving local minima trappings presents a hardest problem, while the second step resorts to Hungarian type algorithms [1–3,9], including the most famous Hungarian algorithm of [10,11], the improved Hungarian algorithm of [12], Auction and RELAX-II algorithms of [13,14], the signature methods of [15,16], or JVC algorithm of [17], all of which have the complexity of $O(N^3)$.

It is an observation that each column and each row of the involved coefficient matrices can be made to form a movable uni-modal series (see Eq. (6) in Section 3, and Definition 1) that has been instrumental in introducing a new rotational angular sorting algorithm. As we prove in this paper, the new algorithm is capable of reducing the $O(N^3)$ complexity of Hungarian type algorithms to $O(N^2)$ when introduced into the second step above.

The rest of the paper will be organized as follows. In Section 2, we review the formulation of the problem and the ML-based relaxation algorithm. The theorems and routine for establishing an optimality of the data association problem are given in Section 3. Section 4 includes some concluding remarks.

2. Problem formulation and ML principle

2.1. Problem formulation

The detailed formulation of the multitarget, multisensor tracking problem could be found in [1], and it is reviewed here only briefly. The following notations are used.

1. $\beta^i(j) = (\beta^{1,i}(j), \beta^{2,i}(j), \dots, \beta^{N,i}(j))'$ denotes the N-tuple measuring bearing data vector of sensor i at time j, where $\beta^{t,i}(j)$ $(0 \le \beta^{t,i}(j) < 2\pi)$ is the measuring bearing data from target t for the sensor i at time j.
2. $\hat{\beta}^i(j) = (\hat{\beta}^{1,i}(j), \hat{\beta}^{2,i}(j), \dots, \hat{\beta}^{N,i}(j))'$ is the cumulative N-tuple bearing estimate measurement vector of sensor i at time j, where $\hat{\beta}^{t,i}(j)$ $(0 \le \hat{\beta}^{t,i}(j) < 2\pi)$ is the estimated measurement for the target t from the sensor i at time j. This can be obtained from the initial target state vector $\hat{X}$ which describes the position and velocity of all the targets, by an arctangent function [1].
3. $C^i(j)$ is a data assignment matrix, whose components consist of 0–1 elements only with just one 1 element appearing in each of the rows and columns where the entry $[C^i(j)]_{tm} = 1$ denotes that the mth element of the measurement vector $\beta^i(j)$ is associated with the tth target. $c(i, j, t)$ is the integer satisfying $[C^i(j)]_{tc(i,j,t)} = 1$.

A solution to the ML-based multitarget tracking algorithm is obtained by maximizing the conditional likelihood of the measurements of the sensors $(\beta^1(0), \beta^1(1), \dots, \beta^1(k), \beta^2(0), \dots, \beta^2(k), \dots, \beta^s(0), \dots, \beta^s(k))$ given $C^k = \{C^1(0), C^1(1), \dots, C^1(k), C^2(0), \dots, C^2(k), \dots, C^s(0), \dots, C^s(k)\}$ and the initial target state vector $\hat{X}$

$$\Lambda = \frac{1}{c}\exp\left\{-\frac{1}{2}\sum_{i=1}^{s}\sum_{j=0}^{k}[C^i(j)\beta^i(j)\dot{-}\hat{\beta}^i(j)]'R_i^{-1}[C^i(j)\beta^i(j)\dot{-}\hat{\beta}^i(j)]\right\}. \quad (1)$$

This is equivalent to minimizing the corresponding ASE,

$$\begin{aligned}\mathrm{ASE} &= \frac{1}{skN}\sum_{i=1}^{s}\sum_{j=0}^{k}[C^i(j)\beta^i(j)\dot{-}\hat{\beta}^i(j)]'R_i^{-1}[C^i(j)\beta^i(j)\dot{-}\hat{\beta}^i(j)] \\ &= \frac{1}{skN}\sum_{t=1}^{N}\sum_{i=1}^{s}\sum_{j=0}^{k}\left(\frac{\beta^{c(i,j,t),i}\dot{-}\hat{\beta}^{t,i}}{\sigma_i}\right)^2. \end{aligned} \quad (2)$$

$R_i = \sigma_i^2 I$ of Eqs. (1) and (2) denotes the $N \times N$ diagonal noise covariance matrix at the ith sensor, and c is a constant independent of C^k and the initial target state vector $\hat{X}$.

Note that, Eqs. (1) and (2) differ distinctly from those of the existing references on this topic [1,18,19] only by a newly introduced operation "$\dot{-}$", which is defined as

$$a\dot{-}b = \begin{cases} a - b, & \text{if } -\pi < a - b \le \pi, \\ a - b - 2\pi, & \text{if } a - b > \pi, \\ a - b + 2\pi, & \text{if } a - b \le -\pi. \end{cases} \quad (3)$$

We see immediately that this new operation is needed to ensure the angular differences formed to be in $(-\pi, \pi]$ such that the mismatching of angles across the so-called Riemann sheet is avoided. In the conventional difference formula based on normal "$-$" [1,4,18,19], this mismatching is inevitable. As we will show in footnote 1, we could in fact develop a faster data assignment algorithm with complexity $O(N \log N)$ for such a conventional formula.

2.2. Outline of ML-based relaxation algorithm for multitarget tracking

The ML-based relaxation method proceeds repeatedly along the following process until the solution converges: for fixed C^k, ASE is minimized with respect to $\hat{X}$. For given $\hat{X}$ (and thus $\hat{\beta}^i(j)$ for all i and j), ASE is minimized with respect to C^k. A standard technique used in these steps is a Gauss–Newton downhill type algorithm for the first process while Hungarian type data assignment algorithms are exploited for the data association process.

It is the second stage of the process where our new rotational sorting-based algorithm replaces the Hungarian type assignment algorithms improving the complexity from $O(N^3)$ of the Hungarian type to $O(N^2)$ for this step. The new algorithm will be shown and proved in Section 3.

3. The data association routine and rotational sort

On writing

$$E_{ij} = \sum_{t=1}^{N} \left(\frac{\beta^{c(i,j,t),i}(j) \dot{-} \hat{\beta}^{t,i}(j)}{\sigma_i} \right)^2, \tag{4}$$

the ASE can be expressed as

$$\text{ASE} = \frac{1}{skN} \sum_{i=1}^{s} \sum_{j=0}^{k} E_{ij}. \tag{5}$$

We see easily that that the minimizing process of ASE with respect to C^k for given $\hat{X}$ is equivalent to minimizing each individual term E_{ij} independently with respect to $C^i(j)$.

This requires a minimization of the following cost function:

$$T = \sum_{n=1}^{N} \sum_{m=1}^{N} \rho_{nm} [C^i(j)]_{nm}, \tag{6}$$

where $\rho_{nm} = (\hat{\beta}_n \dot{-} \beta_m)^2$, where $\beta = (\beta_1, \beta_2, \dots, \beta_N)'$, $\hat{\beta} = (\hat{\beta}_1, \hat{\beta}_2, \dots, \hat{\beta}_N)'$ are the N-tuple real bearing measurement vector and an N-tuple estimated bearing vector, respectively.

Most of the existing publications solve the data association problem in the minimization equation (6) by general Hungarian type algorithms. But we now show that the following uni-modal property of the coefficient matrix $[\rho_{nm}]_{N \times N}$ of Eq. (6) allows a more efficient and simpler algorithm.

Observation: Each column and each row of matrix $[\rho_{nm}]_{N \times N}$, respectively, form a movable uni-modal series, if both of $\hat{\beta}_1, \hat{\beta}_2, \dots, \hat{\beta}_N$, and $\beta_1, \beta_2, \dots, \beta_N$ have been sorted into increasing order.

The definition of a movable uni-modal series is given as follows.

Definition 1. The finite series of real numbers $a_1, a_2, \dots, a_n$, is called a movable uni-modal series if and only if there exists two element a_i and a_j, such that the series $a_i, a_{i+1}, \dots, a_{j-1}$ is a strictly decreasing series and $a_j, a_{j+1}, \dots, a_{i-1}$ is a strictly increasing series, where a_{i+n} is denoted as a_i.

Taking a full advantage of the uni-modality of the series, we will now prove that an rotational sorting algorithm can be used for the data assignment process of multitarget tracking problem, reducing the computational complexity from $O(N^3)$ of the Hungarian type algorithms to $O(N^2)$.

Definition 2. For arbitrary two angles α and β, $0 \leq \alpha, \beta < 2\pi$, $\min\{\alpha, \beta\}$ and $\max\{\alpha, \beta\}$ are defined as

$$\min\{\alpha, \beta\} = \begin{cases} \beta, & \text{if } \alpha \dot{-} \beta \geq 0, \\ \alpha, & \text{otherwise.} \end{cases} \tag{7}$$

$$\max\{\alpha, \beta\} = \begin{cases} \alpha, & \text{if } \alpha \dot{-} \beta \geq 0, \\ \beta, & \text{otherwise.} \end{cases} \tag{8}$$

Definition 3 (Rotational sort). For a list of angles $\alpha_1, \alpha_2, \ldots, \alpha_n$, $0 \leq \alpha_i < 2\pi$, $i = 1, 2, \ldots, n$, the rotational sort of the angles is defined as a permutation $\alpha_{i_1}, \alpha_{i_2}, \ldots, \alpha_{i_n}$ such that $\alpha_{i_u} - \alpha_{i_1} \bmod 2\pi \geq \alpha_{i_v} - \alpha_{i_1} \bmod 2\pi$, for any $1 \leq u < v \leq n$.

It should be important to note that even if $\alpha_1, \alpha_2, \ldots, \alpha_n$ are all distinct and unequal, the rotational sorting of the list is not unique. In fact, there are n rotational sorts in this case. In Fig. 2, $\alpha_3, \alpha_5, \alpha_1, \alpha_2, \alpha_4$ is a rotational sort of the list $\alpha_1, \alpha_2, \alpha_3, \alpha_4, \alpha_5$, satisfying Definition 3.

The notation $\alpha_1 \prec \alpha_2 \prec \cdots \prec \alpha_n$ implies that a list of $\alpha_1, \alpha_2, \ldots, \alpha_n$ has been rotationally sorted. $\alpha_3 \prec \alpha_5 \prec \alpha_1 \prec \alpha_2 \prec \alpha_4$ is a rotationally sorted list for Fig. 2. And for a given rotationally sorted list of angles, we can always use "$\prec$" to denote a sequence of any two angles in the list, if the condition of Definition 3 is satisfied. The following Lemma 1 is needed to prove Theorem 1.

Lemma 1. *Let C be an assignment matrix between vector* $\beta = (\beta_1, \beta_2, \ldots, \beta_N)'$ *and* $\hat{\beta} = (\hat{\beta}_1, \hat{\beta}_2, \ldots, \hat{\beta}_N)'$, *satisfying* $[C]_{j_2 i_1} = [C]_{j_1 i_2} = 1$, *with* $\beta_{i_1} \neq \beta_{i_2}$ *and* $\hat{\beta}_{j_1} \neq \hat{\beta}_{j_2}$. *If we have a rotational sort of the angles* $\beta_{i_1}, \beta_{i_2}, \hat{\beta}_{i_1}$, *and* $\hat{\beta}_{i_2}$, *satisfying property* (1), *property* (2) *or* (2′), *and property* (3) *or* (3′) *below, then C cannot be a best assignment matrix*:

(1) $\hat{\beta}_{j_1} \prec \hat{\beta}_{j_2}$, *and* $\beta_{i_1} \prec \beta_{i_2}$;

(2) $\min\{\beta_{i_1}, \hat{\beta}_{j_1}\} = \beta_{i_1}$, $\hat{\beta}_{j_2} \dot{-} \beta_{i_1} \geq 0$, *and* $\hat{\beta}_{j_2} \dot{-} \hat{\beta}_{j_1} \geq 0$;

(2′) $\min\{\beta_{i_1}, \hat{\beta}_{j_1}\} = \hat{\beta}_{j_1}$, $\beta_{i_2} \dot{-} \hat{\beta}_{j_1} \geq 0$, *and* $\beta_{i_2} \dot{-} \beta_{i_1} \geq 0$;

(3) $\max\{\beta_{i_2}, \hat{\beta}_{j_2}\} = \beta_{i_2}$, $\beta_{i_2} \dot{-} \hat{\beta}_{j_1} \geq 0$, *and* $\hat{\beta}_{j_2} \dot{-} \hat{\beta}_{j_1} \geq 0$;

(3′) $\max\{\beta_{i_2}, \hat{\beta}_{j_2}\} = \hat{\beta}_{j_2}$, $\hat{\beta}_{j_2} \dot{-} \beta_{i_1} \geq 0$, *and* $\beta_{i_2} \dot{-} \beta_{i_1} \geq 0$.

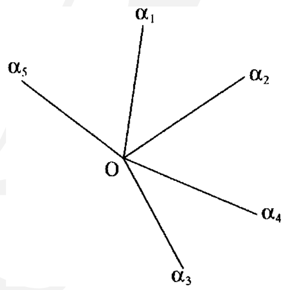


Fig. 2. An example of rotational sorted list.

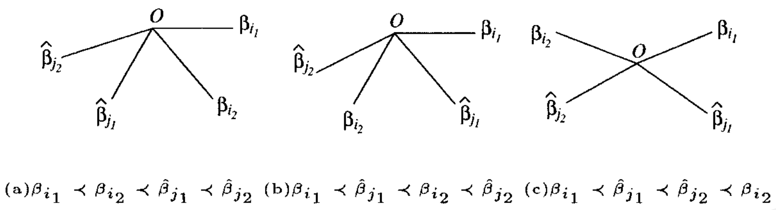


Fig. 3. An example of the four angles in Lemma 1.

Proof. Without loss of generality, we suppose that property (2) is applicable. Then we will need to consider only the three cases of Fig. 3: (a) $\beta_{i_1} \prec \beta_{i_2} \prec \hat{\beta}_{j_1} \prec \hat{\beta}_{j_2}$, (b) $\beta_{i_1} \prec \hat{\beta}_{j_1} \prec \beta_{i_2} \prec \hat{\beta}_{j_2}$ and (c) $\beta_{i_1} \prec \hat{\beta}_{j_1} \prec \hat{\beta}_{j_2} \prec \beta_{i_2}$ separately. For cases of (b) and (c), it is easy to establish that $(\hat{\beta}_{j_2} \dot{-} \beta_{i_2})^2 + (\hat{\beta}_{j_1} \dot{-} \beta_{i_1})^2 < (\hat{\beta}_{j_2} \dot{-} \beta_{i_1})^2 + (\hat{\beta}_{j_1} \dot{-} \beta_{i_2})^2$.

For case (a), we have

$$\begin{aligned}(\hat{\beta}_{j_2} \dot{-} \beta_{i_2})^2 + (\hat{\beta}_{j_1} \dot{-} \beta_{i_1})^2 &= ((\hat{\beta}_{j_2} \dot{-} \beta_{i_1}) - (\beta_{i_2} \dot{-} \beta_{i_1}))^2 + (\hat{\beta}_{j_1} \dot{-} \beta_{i_1})^2 \\ &< (\hat{\beta}_{j_2} \dot{-} \beta_{i_1})^2 + ((\hat{\beta}_{j_1} \dot{-} \beta_{i_1}) - (\beta_{i_2} \dot{-} \beta_{i_1}))^2 \\ &= (\hat{\beta}_{j_2} \dot{-} \beta_{i_1})^2 + (\hat{\beta}_{j_1} \dot{-} \beta_{i_2})^2. \qquad \square \end{aligned} \tag{9}$$

The conclusion of the lemma is now immediate.

Theorem 1. *Let β_{i_1}, β_{i_2} and β_{i_3} be distinct and pair-wise unequal elements of the vector $\beta = (\beta_1, \beta_2, \dots, \beta_N)'$, satisfying $\beta_{i_1} \prec \beta_{i_2} \prec \beta_{i_3}$, and $\hat{\beta}_{j_1}$, $\hat{\beta}_{j_2}$ and $\hat{\beta}_{j_3}$ be distinct and pair-wise unequal elements of the vector $\hat{\beta} = (\hat{\beta}_1, \hat{\beta}_2, \dots, \hat{\beta}_N)'$, satisfying $\hat{\beta}_{j_1} \prec \hat{\beta}_{j_2} \prec \hat{\beta}_{j_3}$. If C is the assignment matrix satisfying $[C]_{j_1 i_1} = [C]_{j_3 i_2} = [C]_{j_2 i_3} = 1$, then C cannot be a best assignment matrix.*

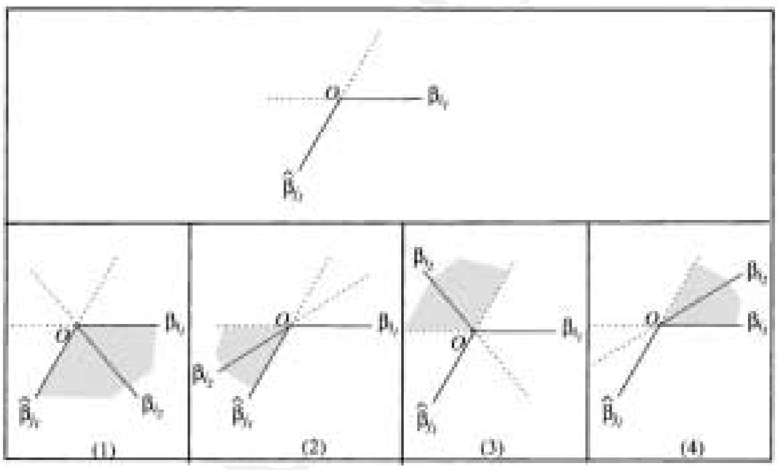


Fig. 4. The different positions for β_{i_2}.

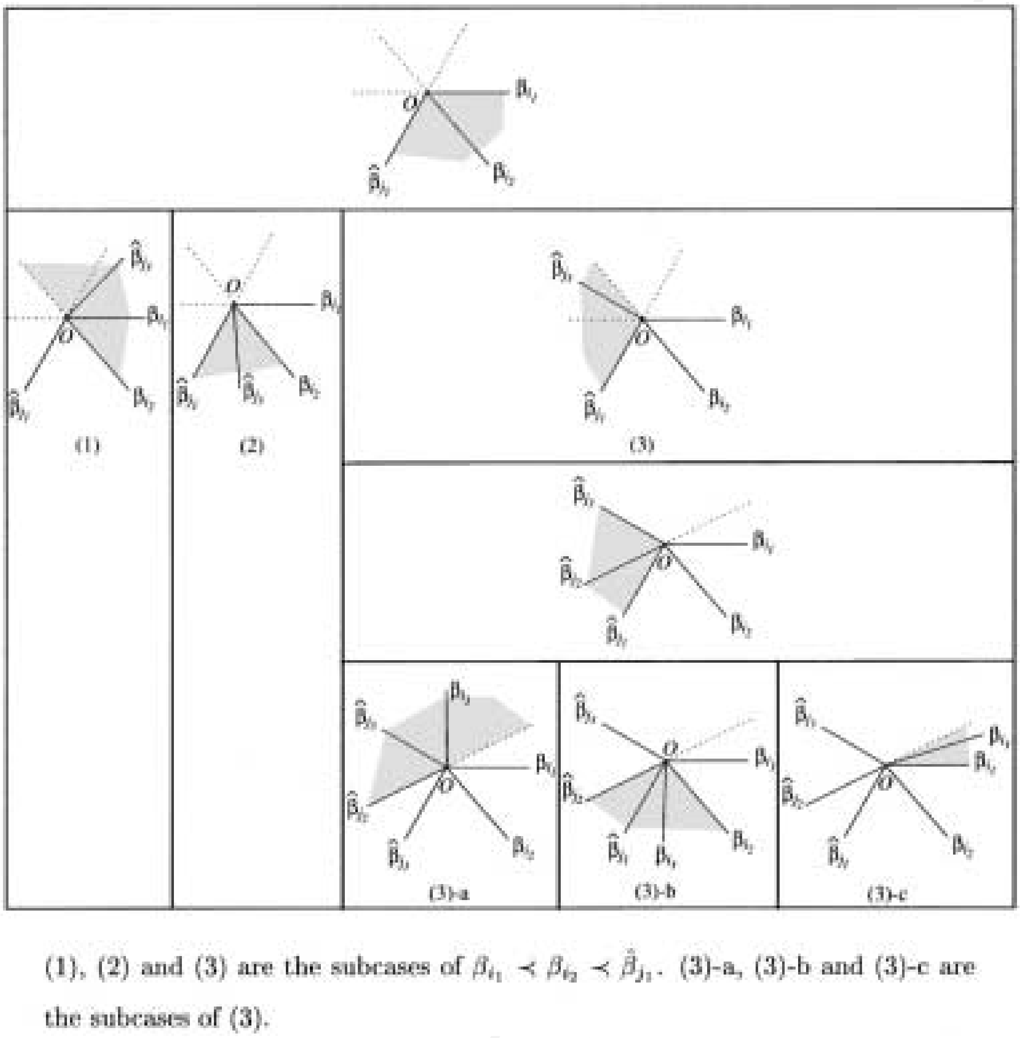
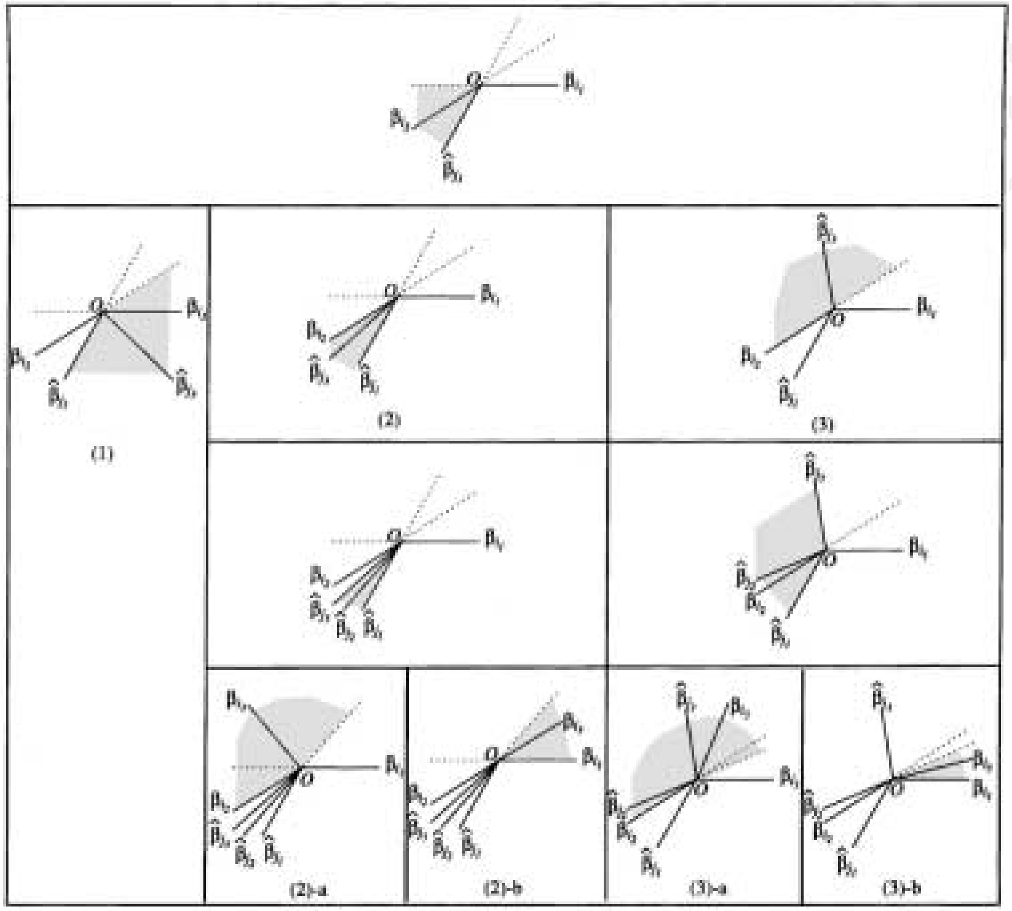


Fig. 5. The case $\beta_{i_1} \prec \beta_{i_2} \prec \hat{\beta}_{j_1}$. (1), (2) and (3) are the subcases of $\beta_{i_1} \prec \beta_{i_2} \prec \hat{\beta}_{j_1}$. (3)-a, (3)-b and (3)-c are the subcases of (3).

Proof. Without loss of generality, we assume that $\hat{\beta}_{j_1} \dot{-} \beta_{i_1} > 0$ (see Fig. 4). For an angle α in $0 \le \alpha < 2\pi$, we denote $-\alpha$ as $\pi + \alpha$, if $0 \le \alpha < \pi$, and $\alpha - \pi$, otherwise. To determine a possible position for β_{i_2}, we can enumerate the following four cases of Fig. 4(1)–(4), which represent the cases of $\beta_{i_1} \prec \beta_{i_2} \prec \hat{\beta}_{j_1}$, $\hat{\beta}_{j_1} \prec \beta_{i_2} \prec -\beta_{i_1}$, $-\beta_{i_1} \prec \beta_{i_2} \prec -\hat{\beta}_{j_1}$ and $-\hat{\beta}_{j_1} \prec \beta_{i_2} \prec \beta_{i_1}$, respectively. We examine each of the four cases separately.

For the first case of $\beta_{i_1} \prec \beta_{i_2} \prec \hat{\beta}_{j_1}$, we further consider the subcases depending on the positions of $\hat{\beta}_{j_3}$ (see Fig. 5). It is easy to prove the theorem from Lemma 1 for this case. C cannot be a best assignment for the cases of Fig. 5(1) and (2), because $[C]_{j_1 i_1} = [C]_{j_3 i_2} = 1$; the case of Fig. 5(3) must be subdivided into the subcases of Fig. 5(3)-a, (3)-b and (3)-c, where $[C]_{j_3 i_2} = [C]_{j_2 i_3} =$



(1), (2) and (3) are the subcases of $\hat{\beta}_{j_1} \prec \beta_{i_2} \prec -\beta_{i_1}$. (2)-a and (2)-b are the subcases of (2), (3)-a and (3)-b are the subcases of (3).

Fig. 6. The case $\hat{\beta}_{j_1} \prec \beta_{i_2} \prec -\beta_{i_1}$. (1), (2) and (3) are the subcases of $\hat{\beta}_{j_1} \prec \beta_{i_2} \prec -\beta_{i_1}$. (2)-a and (2)-b are the subcases of (2), and (3)-a and (3)-b are the subcases of (3).

1, $[C]_{j_3 i_2} = [C]_{j_2 i_3} = 1$ and $[C]_{j_1 i_1} = [C]_{j_2 i_3} = 1$ support the evidence of not being optimal separately.

For the remaining cases of $\hat{\beta}_{j_1} \prec \beta_{i_2} \prec -\beta_{i_1}$, $-\beta_{i_1} \prec \beta_{i_2} \prec -\hat{\beta}_{j_1}$ and $-\hat{\beta}_{j_1} \prec \beta_{i_2} \prec \beta_{i_1}$ of Figs. 6–8, respectively, we could always find a piece of evidence in each case that C is not the best according to Lemma 1. □

The following theorem shows that the sorting-based routine of $O(N^2)$ complexity gives an optimal assignment matrix, minimizing the ASE for a fixed $\hat{X}$.

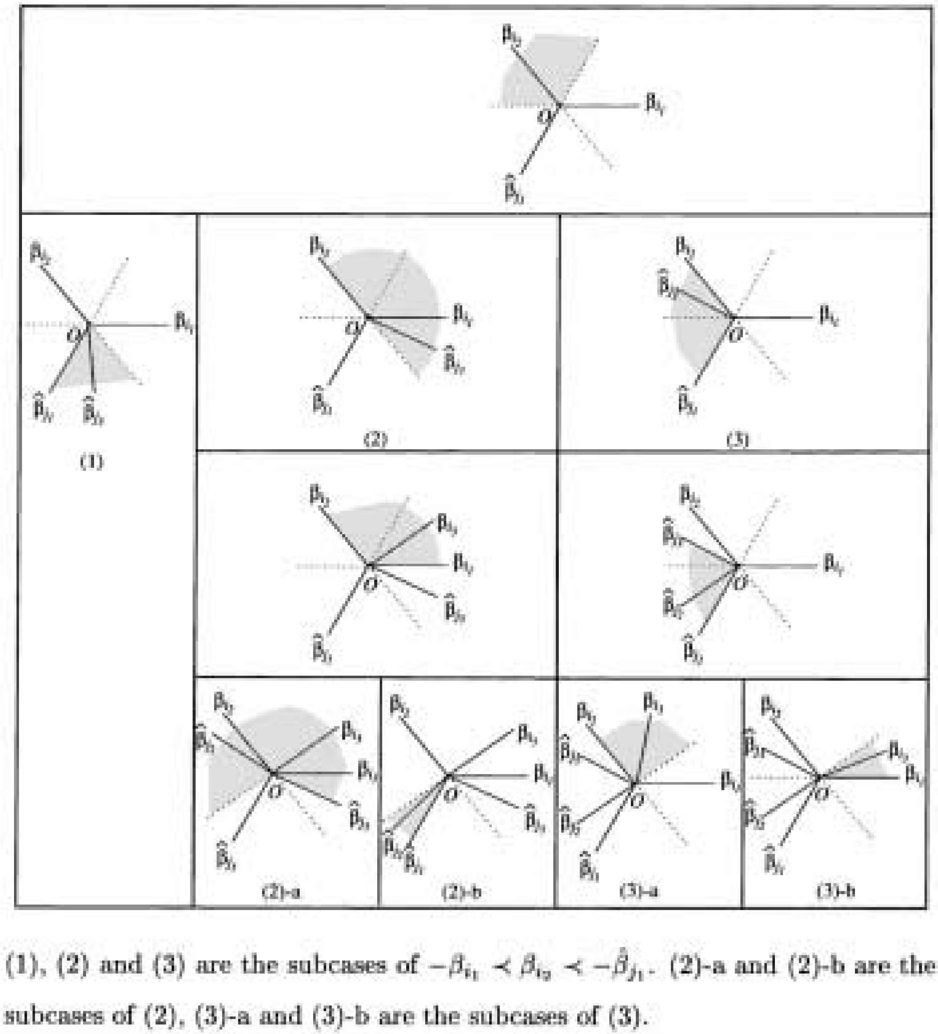
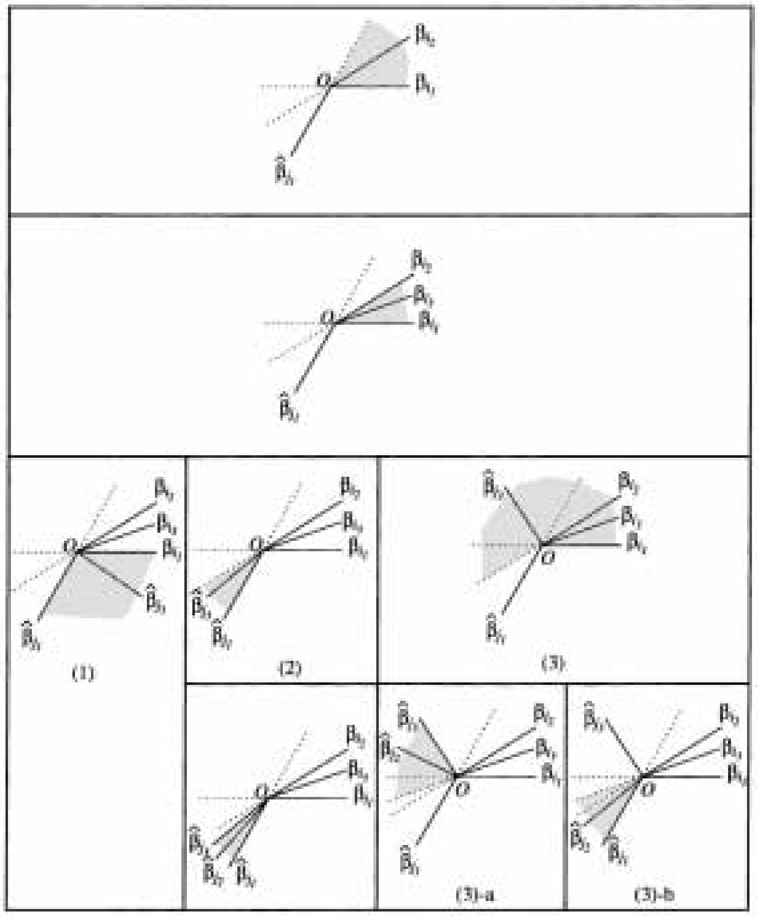


Fig. 7. The case $-\beta_{i_1} \prec \beta_{i_2} \prec -\hat{\beta}_{j_1}$. (1), (2) and (3) are the subcases of $-\beta_{i_1} \prec \beta_{i_2} \prec -\hat{\beta}_{j_1}$. (2)-a and (2)-b are the subcases of (2), and (3)-a and (3)-b are the subcases of (3).

Theorem 2. *Suppose that the cumulative bearing estimate measurement vectors for sensor* i *at time* j $(0 \le j \le k)$ *is given by* $\hat{\beta}^i(j) = (\hat{\beta}^{1,i}(j), \hat{\beta}^{2,i}(j), \dots, \hat{\beta}^{N,i}(j))'$, *and that the real bearing data of sensor* i *is given by* $\beta^i(j) = (\beta^{1,i}(j), \beta^{2,i}(j), \dots, \beta^{N,i}(j))'$. *Given two* 0–1 *distinct matrices* C_1 *and* C_2, *each having just one* 1 *element in each row and each column, such that* $C_1\beta^i(j) = (a_1, a_2, \dots, a_N)'$, $C_2\hat{\beta}^i(k) = (b_1, b_2, \dots, b_N)'$, *where* $a_1 \prec a_2 \prec \cdots \prec a_N$ *and* $b_1 \prec b_2 \prec \cdots \prec b_N$, *then the best data assignment matrix* $C^i(j)$ *for minimizing ASE with fixed* $\hat{X}$ *should be chosen from the set* $\{C_2' P_r C_1 | P_r = [p_{r_1 r_2}]_{N\times N}, r = 0, 1, 2, \dots, N-1\}$, *where* p_{r_1,r_2} *is defined as*

$$p_{r_1 r_2} = \begin{cases} 1, & \text{if } r_2 - r - r_1 \equiv 0 \bmod N, \\ 0, & \text{otherwise.} \end{cases} \tag{10}$$



(1), (2) and (3) are the subcases of $-\hat{\beta}_{j_1} \prec \beta_{i_2} \prec \beta_{i_1}$. 3-(a) and (3)-b are the subcases of (3).

Fig. 8. The case $-\hat{\beta}_{j_1} \prec \beta_{i_2} \prec \beta_{i_1}$. (1), (2) and (3) are the subcases of $-\hat{\beta}_{j_1} \prec \beta_{i_2} \prec \beta_{i_1}$. 3-(a) and (3)-b are the subcases of (3).

Proof. If $C_r^i(j) = C_2' P_r C_1$, we have

$$\mathrm{ASE} = \frac{1}{skN}\sum_{i=1}^{s}\sum_{j=0}^{k} E_{ij}(r). \tag{11}$$

We write $E_{ij}(r) = [C^i(j)\beta^i(j) \dot{-} \hat{\beta}^i(j)]' R_i^{-1} [C^i(j)\beta^i(j) \dot{-} \hat{\beta}^i(j)]$. Noting that $C_2' = C_2^{-1}$, we have the following relation:

Table 1
Routine for finding an optimal assignment matrix

1. Sort the terms in $\beta^i(j)$ into increasing order as $\beta^{e_1,i}(j), \beta^{e_2,i}(j), \dots, \beta^{e_N,i}(j)$
2. Sort the terms in $\hat{\beta}^i(j)$ into increasing order as $\hat{\beta}^{f_1,i}(j), \hat{\beta}^{f_2,i}(j), \dots, \hat{\beta}^{f_N,i}(j)$
3. Construct C_1 for (1) such that $[C_1]_{te_t} = 1$ for each $1 \le t \le N$
4. Construct C_2 for (2) such that $[C_2]_{tf_t} = 1$ for each $1 \le t \le N$
5. For each P_r, $r = 0, 1, \dots, N-1$, calculate $C^i(j) = C_2' P_r C_1$ and the associated $E_{ij}(r)$, then choose the best one

$$\begin{aligned} E_{ij}(r) &= [C_2' P_r C_1 \beta^i(j) \dot{-} \hat{\beta}^i(j)]' R_i^{-1} [C_2' P_r C_1 \beta^i(j) \dot{-} \hat{\beta}^i(j)] \\ &= [P_r C_1 \beta^i(j) \dot{-} C_2 \hat{\beta}^i(j)]' R_i^{-1} [P_r C_1 \beta^i(j) \dot{-} C_2 \hat{\beta}^i(j)] = \frac{1}{\sigma_i} \sum_{t=1}^{N} (a_t \dot{-} b_{t+r-1 \bmod N+1})^2. \end{aligned} \quad (12)$$

We should note that the set $\{C_2' P_r C_1 | r = 0, 1, 2, \dots, N-1\}$ has been obtained after enumerating all the following matches:

$$\begin{pmatrix} \beta^{1,i}(j) & \beta^{2,i}(j) & \cdots & \beta^{N,i}(j) \\ \updownarrow & \updownarrow & & \updownarrow \\ \hat{\beta}^{1,i}(j) & \hat{\beta}^{2,i}(j) & \dots & \hat{\beta}^{N,i}(j) \end{pmatrix}, \begin{pmatrix} \beta^{1,i}(j) & \beta^{2,i}(j) & \cdots & \beta^{N,i}(j) \\ \updownarrow & \updownarrow & & \updownarrow \\ \hat{\beta}^{N,i}(j) & \hat{\beta}^{1,i}(j) & \dots & \hat{\beta}^{N-1,i}(j) \end{pmatrix}, \dots, \begin{pmatrix} \beta^{1,i}(j) & \beta^{2,i}(j) & \cdots & \beta^{N,i}(j) \\ \updownarrow & \updownarrow & & \updownarrow \\ \hat{\beta}^{2,i}(j) & \hat{\beta}^{3,i}(j) & \dots & \hat{\beta}^{1,i}(j) \end{pmatrix}.$$

The conclusion of Theorem 2 is immediate from Theorem 1. □

2 The theorem shows that an optimal local matrix $C^i(j)$ for a fixed $\hat{X}$ can be obtained first by sorting $\beta^i(j)$ and $\hat{\beta}^i(j)$ by any of the sorting algorithm with the complexity of $O(N \log N)$, and then choosing the one satisfying Theorem 2 from among N potential candidates. This can be obviously done with the complexity of $O(N^2)$. This is illustrated in the new procedural flow of Table 1.[1] Note that the sorting of $\hat{\beta}^i(j)$ could be done at the very beginning of an iteration whose order will remain valid at each step of the iterations. The complexity of computing the present assignment matrix $C^i(j)$ is far more efficient than the Hungarian method type algorithms used by most of previous works (e.g. [1,4]), including a Hungarian algorithm of [10,11], an improved Hungarian algorithm of [12], Auction and RELAX-II algorithms of [13,14], or the signature methods of [15,16], all having the complexity of $O(N^3)$.

4. Conclusion

Taking full advantage of the uni-modality of the series in the coefficients of the related data association matrix, we have developed a new data association algorithm problem embedded in the ML-based relaxation solution to a multitarget tracking problem, reducing the complexity from $O(N^3)$ of the conventional Hungarian type routines to $O(N^2)$.

[1] For the particular case of "$\dot{-}$" being "$-$", it is easy to see that the routine of Table 1 can be simplified to complexity of $O(N \log N)$, since $C^i(j) = C_2' C_1$ gives the best assignment without an enumeration of P_r of the present case.

The theorems we have proved depend on the assumption of no cluttering and no missing measuring data. Although the no cluttering assumption may not be too unrealistic in many of the air traffic surveillance system [20], the no missing data assumption may be too restrictive in practice. We hope to extend the analysis to take the missing data into consideration, hoping to obtain the algorithm of lower complexity, if only the missing data is considered but not the cluttering.

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