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AN EFFICIENT COMPUTATIONAL APPROACH FOR MULTITARGET TRACKING FROM BEARINGS-ONLY MEASUREMENTS BY DECENTRALIZED COOPERATIVE PROCESSING

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Abstract. *This paper firstly has proved that the well known Hungarian type assignment algorithms* [14, 4] *embedded in the relaxation-based maximum-likelihood (ML) solution for a bearings-only passive multitarget-multisensor tracking problem can be replaced by a much simpler sorting algorithm of* $O(N \log N)$ *complexity, provided that the sensor system is ideal such that the system has no cluttering points nor missing data. A new computationally efficient ML-based relaxation method for multitarget motion analysis under a fixed networked multisensor environment is then developed by exploiting a new cooperative decentralized processing scheme of* [15]. *Embedding locally an optimal data association algorithm of* $O(N \log N)$ *into each of Gauss-Newton's downhill iteration loops, our simulations show that we are able to track multiple targets with improved accuracy and efficiency, where all targets are allowed to move in variable directions at varying speeds. The solution we have developed constitutes a suboptimal solution in the sense of* [14, 8] *because an optimal solution is embedded within part of the entire optimization problem.*

keywords: Hungarian Algorithm, data associate problem, multitarget motion tracking, bearings-only measurement, distributed passive sensor network, cooperative processing

1. Introduction.

Multitarget tracking from bearings-only measurements under a distributed sensor network is a hard inverse problem [14], largely due to uncertainty in data assignment problem of tracks with respect to the multitarget objects whose time dependent motions are entirely independent and are often subject to components of independent noise. In spite of extensive research work on the subject [18, 12, 10], the problem remains unsolved because it is shown that the statistic data assignment problems with more than three sensors is an NP hard problem [5, 6, 11]. The difficulty increases perhaps beyond an exponential complexity if the number of targets, the number of sensors and the number of sampling scans increase [14, 8] with notable exceptions of the case studied by Zhou and Bose [18], where targets can be properly grouped into clusters so that the associate problem can be decomposed into smaller subproblems.

Among all the methods applied to multitarget tracking analysis, a maximum likelihood principle (ML)-based relaxation method is most commonly used. The relaxation method exploits in fact a gradient-search based approach on batch processing of all the available measurements. It involves the iteration loops for two major processes; one for maximizing a likelihood function (LF) with respect to initial states such as positions, velocities of targets with the data association between the bearing measurements and targets fixed, and the other for maximizing the LF with respect to data association with the initial states of targets fixed. The first procedure is most frequently implemented by a Gauss-Newton gradient method [5], while the second is formulated as a 2-D assignment problem often solved by Hungarian type algorithms or by an improved Lagrangian relaxation as a series of relaxed 2-D assignment problems [12]. The Hungarian type algorithms for the 2-D assignment problem includes the original Hungarian algorithm of [7], an improved Hungarian algorithm of [4], Auction and RELAX-II algorithms of [2], and the signature methods of [1], all of which are of $O(\min\{NM^2, N^2M\})$ in complexity, where N and M are the number of targets and number of bearing measurement data respectively.

Yet, as is true with most of NP hard problems, the ML-based relaxation methods for the multitarget tracking problem encounter severe difficulties firstly by unnecessarily complex algorithms which are most easily trapped by local minima. Much progress has been reported in resolving a local minimum entrapping problem including methods of simulated annealing, and neural computing to help obtain a global minimum [14, 8].

A decentralized cooperative search [10] offers a new approach to obtaining a global minimum in multidimensional space having multiple-local minimal functions. Unlike a centralized processing system where all of the sensor measured data are processed by a single central processor, in the decentralized cooperative approach [16, 17], all bearing measured data are processed by a multiple of local processors, where each processor estimates multitarget tracking by exchanging their intermediate estimated results obtained by other processors when needed. The centralized single processor processing scheme encounters not only an instability in computation due to the presence of widely scattered local minima over the solution space but also due to an additional cost expected in data transfer. How to find a global minimum without being trapped by local minima forms the core of the solution methods of the problem. Under multisensor and multitarget tracking, we do not need to have everything done by a single, centralized computer and the concentrated computing may be costly in the end. The decentralized cooperative search of this paper on the other hand exploits the multiple processor scheme as with the multiple sensor environment where each processor has its own smaller search space and when necessary, they cooperatively search for a global minimum over the multiply branched solution space in parallel.

In this paper, the multitarget multisensor tracking problem is formulated by the decentralized cooperative computational scheme [10] where multiple processors cooperate. As in most of the literature on the subject, we obtain an optimal solution from bearing measurement data of multiple targets by maximizing the most popular ML(maximum likelihood) principle-based conditional probabilities. Our basic strategy in computation is the following: We first observe and then prove the theorem that an optimal minimum error assignment matrix can be most easily found if the bearings in position of the targets are known and sorted in magnitude. Encouraged by the theorem, we obtain a suboptimal solution in the sense of [14, 8] by fusing the locally optimal data assignment result with locally valid minimal least square errors until results from all processors converge. A solution is called a suboptimal one if an optimal solution is embedded within part of the entire optimization problem [14, 8]. We develop a new rapidly converging relaxation algorithm for each processor in the decentralized processing scheme by embedding the assignment matrices into the loops of Gauss-Newton downhill iterations without increasing much of the complexity. We have tested the algorithm proposed by simulating the tracks of as many as 8 and 12 targets with 4 and 6 processors respectively.

The remaining sections of the paper are arranged as follows: Section 2 reviews the formulation of the problem based on the related ML relaxation approach, while section 3 shows how our sorting based algorithm can improve the data assignment problem of the ML relaxation approach under an ideal situation of no missing data and no clutter. The cooperative decentralized processing scheme based on our sorting based data assignment algorithm is provided in section 4. A simulation which verifies the correctness of our algorithm is given in section 5. The concluding remarks are given in section 6.

2. **Problem Formulation and ML Based Relaxation Algorithm .**

2.1. **Problem Formulation .** Following the work of Ting and Iltis [14], a typical multitarget-multisensor encounter can be shown in Fig.1, where each sensor consists of a passive array of acoustic sensors and a front-end direction-of-arrival (DOA) estimator. The positions and velocities of the targets are estimated by finding the set of targets that generates bearing histories that best match the bearing measurements from sensors.

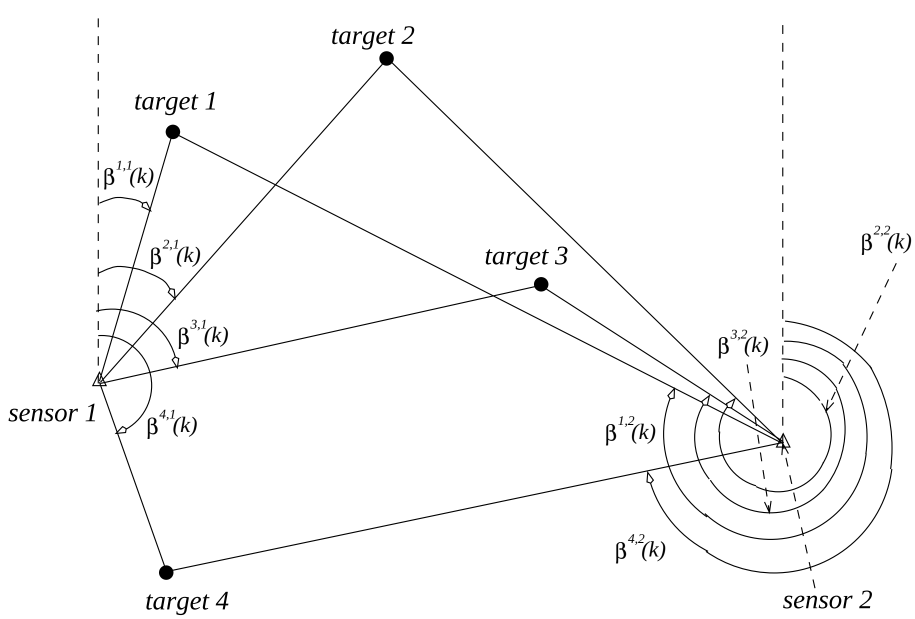


FIGURE 1. Typical Bearings-only Passive Multitarget-Multisensor Encounter

We assume that there are N targets in the surveillance region, and s fixed bearing-only sensors in the plane. The state of a target t at time index j is described by

$$X^t(j) = (r_x^t(j), r_y^t(j), v_x^t(j), v_y^t(j))'$$

where $(r_x^t(j), r_y^t(j))$ denote its Cartesian coordinates and $(v_x^t(j), v_y^t(j))$ its velocity components. The sensor i is located at the positions (r_{xs}^i, r_{ys}^i), $i = 1, 2, \cdots, s$. We assume that at time j, the measurement bearing data from target t for the sensor i can be written using the state vector $X^{t,i}$ as:

$$\beta^{t,i}(j) = \tan^{-1}\left[\frac{r_x^{t,i}(j)}{r_y^{t,i}(j)}\right] + w^{t,i}(j), 0 \le \beta^{t,i}(j) < 2\pi.$$

where $r_x^{t,i}(j) = r_x^t(j) - r_{xs}^i$, $r_y^{t,i}(j) = r_y^t(j) - r_{ys}^i$ and $w^{t,i}(j)$'s denote noise components of the ith sensor all of which are assumed to be white, Gaussian noises with mean zero and variance σ_i^2.

The measuring bearing data vector of sensor i at time j forms a N-tuple vector as

$$\beta^i(j) = (\beta^{1,i}(j), \beta^{2,i}(j), \cdots, \beta^{N,i}(j))'.$$

Because we do not have a priori knowledge on the origin of each measurement, we associate each measurement vector $\beta^i(j)$ with an $N \times N$ data assignment matrix $C^i(j)$, whose components consist of 0-1 elements only with just one 1 element appearing in each of the rows and columns. The entry $[C^i(j)]_{tm} = 1$ denotes that the mth element of the measurement vector $\beta^i(j)$ is associated with the tth target.

Our aim is to track the positions of the tracks $(r_x^t(j), r_y^t(j))$ for all targets t, $t = 1, 2, \cdots, N$, and all the time indexes j, $j = 0, 1, \cdots, K$ from the multisensor measuring data. In our formulation, we allow all the targets to move toward any directions moving at varying velocities. We try to estimate the tracks of targets just over a relatively short period of time, say, p time indexes. During such a short time period, say from time k_1 to time $k_1 - 1 + p$, we may assume that the targets can be considered as moving towards fixed directions at a fixed speed. In the following analysis, the time period during which we track the targets is in fact overlapped, thus for each target t, $(r_x^t(j), r_y^t(j))$ is always computed for several times and averaged values are used as the final result.

2.2. **ML Principle.** To estimate the tracks of targets during a linear period say from time k_1 to k_2 ($k_2 - k_1 + 1 = p$), we need in fact to estimate the initial state for each of the targets $X^t(j)$ at time index $j = k_1$ and the assignment matrixes $C^i(j)$ for $i = 1, 2, \cdots s$ and $j = k_1, k_1 + 1, \cdots, k_2$. Thus we seek a ML solution for this problem.

Given an estimated initial state vector for N targets by $\hat{X}(k_1) = (\hat{X}^1(k_1)', \hat{X}^2(k_1)', \cdots, \hat{X}^N(k_1)')$, we seek the cumulative bearing estimate measurement vector at time j ($k_1 \le j \le k_2$) denoted by

$\hat{\beta}^i(j, \hat{X}_{k_1}) =$

$$\left(\hat{\beta}^{1,i}(j, \hat{X}_{k_1}), \hat{\beta}^{2,i}(j, \hat{X}_{k_1}), \cdots, \hat{\beta}^{N,i}(j, \hat{X}_{k_1})\right)'$$

where

$$\hat{\beta}^{t,i}(j, \hat{X}_{k_1}) = \tan^{-1}\left[\frac{\hat{r}_x^{t,i}(j)}{\hat{r}_y^{t,i}(j)}\right].$$

Denote $C^{p,k_1} = \{C^1(k_1), C^1(k_1+1), \cdots, C^1(k_2), C^2(k_1), \cdots, C^2(k_2), \cdots, C^s(k_1), \cdots, C^s(k_2)\}$ and $\beta^{p,k_1} = (\beta^1(k_1),\ \beta^1(k_1+1), \cdots, \beta^1(k_2),\ \beta^2(k_1),\ \cdots, \beta^2(k_2), \cdots, \beta^s(k_1), \cdots,\ \beta^s(k_2))$, the conditional likelihood of β^{p,k_1} given C^{p,k_1} and $\hat{X}_{k_1}$ can be given by

$$\Lambda\left(\beta^{p,k_1}|C^{p,k_1}, \hat{X}_{k_1}\right) =$$

$$\frac{1}{c}\exp\{-\frac{1}{2}\sum_{i=1}^{s}\sum_{j=k_1}^{k_2}[C^i(j)\beta^i(j) - \hat{\beta}^i(j, \hat{X}_{k_1})]' R_i^{-1}[C^i(j)\beta^i(j) - \hat{\beta}^i(j, \hat{X}_{k_1})]\} \qquad (1)$$

where $R_i = \sigma_i^2 I$ is the $N \times N$ diagonal noise covariance matrix at the ith sensor, and c is a constant independent of C^{p,k_1} and $\hat{X}_{k_1}$. To obtain a solution, we want to maximize Λ by minimizing the corresponding average square error ASE given by

$$\begin{aligned} \text{ASE} &= \frac{1}{spN}\sum_{i=1}^{s}\sum_{j=k_1}^{k_2}[C^i(j)\beta^i(j) - \hat{\beta}^i(j, \hat{X}_{k_1})]' R_i^{-1} \cdot [C^i(j)\beta^i(j) - \hat{\beta}^i(j, \hat{X}_{k_1})] \\ &= \frac{1}{spN}\sum_{t=1}^{N}\sum_{i=1}^{s}\sum_{j=k_1}^{k_2}\left(\frac{\beta^{c(i,j,t),i} - \hat{\beta}^{t,i}}{\sigma_i}\right)^2 \end{aligned}$$

where $c(i,j,t)$ is the integer satisfying $[C^i(j)]_{tc(i,j,t)} = 1$. Write

$$E_t = \sum_{i=1}^{s}\sum_{j=k_1}^{k_2}\left(\frac{\beta^{c(i,j,t),i}(j) - \hat{\beta}^{t,i}(j)}{\sigma_i}\right)^2$$

we have

$$ASE = \frac{1}{skN}\sum_{t=1}^{N} E_t.$$

3. **Sorting Based Assignment Algorithm.** The following theorem shows that the Hungarian assignment algorithm can be replaced by an optimal $O(N\log N)$ complexity of the sorting algorithm as along as local analyses of some given sensor are concerned.

Theorem Suppose that the cumulative bearing estimate measurement vectors for sensor i at time j $(k_1 \le j \le k_2)$ is given by $\hat{\beta}^i(j, \hat{X}_{k_1}) = (\hat{\beta}^{1,i}(j, \hat{X}_{k_1}), \hat{\beta}^{2,i}(j, \hat{X}_{k_1}), \cdots, \hat{\beta}^{N,i}(j, \hat{X}_{k_1}))'$, and that the real bearing data of sensor i is given by $\beta^i(j) = (\beta^{1,i}(j), \beta^{2,i}(j), \cdots, \beta^{N,i}(j))'$. Given two $0-1$ distinct matrices C_1 and C_2, each having just only one 1 element in every row and column of the matrices, such that $C_1 \cdot \beta^i(j) = (a_1, a_2, \cdots, a_N)'$, $C_2 \cdot \hat{\beta}^i(k, \hat{X}_{k_1}) = (b_1, b_2, \cdots, b_N)'$, where $a_1 \le a_2 \le \cdots \le a_N$ and $b_1 \le b_2 \le \cdots \le b_N$, then the $C^i(j) = C_2' C_1$ is a best data assignment matrix for minimizing E for fixed $\hat{X}_{k_1}$.

Proof. Writing $E(C^i(j)) = [C^i(j)\beta^i(j) - \hat{\beta}^i(j, \hat{X}_{k_1})]' R_i^{-1}[C^i(j)\beta^i(j) - \hat{\beta}^i(j, \hat{X}_{k_1})]$, we have

$$ASE = \frac{1}{spN}\sum_{t=1}^{N}\sum_{j=k_1}^{k_2} E(C^i(j)).$$

Noting that $C_2' = C_2^{-1}$, we have the following relation:

$$\begin{aligned} E(C^i(j)) &= [C_2'C_1\beta^i(j) - \hat{\beta}^i(j,\hat{X}_{k_1})]'R_i^{-1}\cdot[C_2'C_1\beta^i(j) - \hat{\beta}^i(j,\hat{X}_{k_1})] \\ &= [C_1\beta^i(j) - C_2\hat{\beta}^i(j,\hat{X}_{k_1})]'R_i^{-1}\cdot[C_1\beta^i(j) - C_2\hat{\beta}^i(j,\hat{X}_{k_1})] \\ &= \frac{1}{\sigma_i}\sum_{t=1}^{N}(a_t - b_t)^2. \end{aligned}$$

To show that $C^i(j) = C_2'C_1$ is the best matrix possible, consider any other assignment matrix, say, $C_0^i(j)$, where m is the smallest integer such that $[C_2'C_0^i(j)]_{mm} \neq 1$, with $[C_2'C_0^i(j)]_{mt_1} = 1$ and $[C_2'C_0^i(j)]_{t_2m} = 1$. It is now easy to construct a new assignment matrix $C_1^i(j)$ such that

$$[C_2'C_1^i(j)]_{jt} = \begin{cases} 1 & \textbf{if } \text{j=t=m} \\ & \quad \text{or } (j = t_2 \text{ and } t = t_1), \\ 0 & \textbf{if } (j = t_2 \text{ and } t = m) \\ & \quad \text{or } (t = t_1 \text{ and } j = m), \\ [C_2'C_0^i(j)]_{jt} & \textbf{otherwise}. \end{cases}$$

Noting that $(a_m - b_m)^2 + (a_{t_1} - b_{t_2})^2 \leq (a_m - b_{t_2})^2 + (a_{t_1} - b_m)^2$ for $t_1 > m$ and $t_2 > m$, we immediately have $E(C_1^i(j)) \leq E(C_0^i(j)$. The theorem follows immediately by mathematical induction. □

The theorem shows that an optimal local matrix $C^i(j)$ can be obtained by sorting $\beta^i(j)$ and $\hat{\beta}^i(j,\hat{X}_{k_1})$ for fixed $\hat{X}_{k_1}$ which requires a remarkably efficient $O(N\log N)$ computing time, offering a new procedural flow of Table 1. The complexity of computing the present assignment matrix $C^i(j)$ is far more efficient than the Hungarian type 2-D data assignment algorithms ([7, 4, 1]) whose computational complexities are all of $O(N^3)$.

TABLE 1. The Procedure for Finding A Best Assignment Matrix

Procedure (Find A Best Assignment Matrix)
Set a suitable target initial state vector $\hat{X}_{k_1}$;
(1) Sort the terms in β_j^i into increasing order as $\beta_{i,j}^{h_1}, \beta_{i,j}^{h_2}, \cdots, \beta_{i,j}^{h_N}$);
(2) Sort the terms in $\hat{\beta}_j^i$ into increasing order as $\hat{\beta}_{i,j}^{f_1}, \hat{\beta}_{i,j}^{f_2}, \cdots, \hat{\beta}_{i,j}^{f_N}$;
(3) Construct M_1 for (1) such that $[M_1]_{h_t i} = 1$ for each $1 \leq t \leq N$;
(4) Construct M_2 for (2) such that $[M_2]_{f_t i} = 1$ for each $1 \leq t \leq N$;
(5) $C_j^i = M_2'M_1$.

4. **Cooperative Computing Algorithm.**

4.1. **Outline of Existing Methods.** Whether centralized or decentralized, the ML-based relaxation methods proceed along the following algorithm: For fixed C^{p,k_1}, ASE is minimized with respect to $\hat{X}_{k_1}$ by a Gauss-Newton relaxation method. For given $\hat{X}_{k_1}$, minimizing ASE with respect to C^{p,k_1} is equivalent to minimizing each individual term independently with respect to $C^i(j)$, and this can be done by an improved Hungarian algorithm of [4]. The ML-based centralized cooperative computing relaxation methods always consists of the two steps of iteration loops.

TABLE 2. New Decentralized Relaxation Method

Procedure (New Decentralized)

Set up a suitable initial state vector $\hat{X}_{k_1}$ for targets;
repeat
1. Given $\hat{X}_{k_1}$, find a set of minimum error assignment matrices C^{p,k_1} for ASE_G;
2. **Repeat**
   2.1. Fix C^{p,k_1}, according to Gaus-Newton downhill principle, go one step with respect to $\hat{X}_{k_1}$;
   2.2. Fix $\hat{X}_{k_1}$, find a set of minimum error assignment matrices C^{p,k_1};
   Until converged
3. Send the initial state vectors of all targets to all other processors;
4. Receive initial state vectors of all targets sent by all other processors;
5. With initial state vectors of all the targets from each sensor s, calculate and choose a best vector giving minimal ASE together with all the best initial state vectors of each sensor from the initial state vectors of all the sensors;

until converged.

For the decentralized cooperative computing case, the system exploits several independent processors but in a cooperative manner; sensors are grouped into several clusters allowing always the overlapping of some sensor with the other clusters and each sensor cluster has its own processor. Each processor estimates multitarget tracking using bearing measurement data collected from local sensors. The processors cooperate and exchange intermediate estimates of target initial vectors with all other processors once in an iteration, then choose a best estimation to provide a least ASE.

4.2. **New Decentralized Computing .** Suppose that for a sensor group G, the indexes of the sensors in the group forms the set G, a local ASE function is defined as

$$\text{ASE}_G = \frac{1}{|G|pN} \sum_{t=1}^{N} \sum_{i \in G} \sum_{j=k_1}^{k_2} \left(\frac{\beta^{c(i,j,t),i}(j) - \hat{\beta}^{t,i}(j)}{\sigma_i} \right)^2$$

Our sorting based data assignment algorithm leads to the new cooperative computing algorithm given in Table 2.

Note that, in step 2.1, the Guass-Newton's downhill algorithm can be found in the Appendix A of [14]. The step 2.2, could be done by the procedure described in Table 1.

The procedure in Tabel 2 computes the tracks of targets in one linear period, i.e. from time k_1 to k_2. A complete algorithm computing the multitarget motion analysis over the entire time period, from time index 0 to K, will be described in Table 3.

5. **Simulation.** The highlight of the present paper is to demonstrate that the originally 2-D data assignment problem often solved by $O(N^3)$ Hungarian type algorithms can solved by a far simpler $O(N \log N)$ 1-D sorting based algorithm. We now demonstrate a numerical simulation of the multitarget tracking problem for a general shape of curved tracks of targets where the data assignment problem constitutes one part of the iteration loops of the entire relaxation processes.

TABLE 3. Algorithm for Tracking Targets

1. Given a suitable initial target state vector $\hat{X}_1$;
2. For $k_1 = 1$ to $k_1 = K - p + 1$ do
   2.1. Use Decentralized Computing Procedure to find a best solution with respect to ASE_G for a time period from k_1 to k_2;
   2.2. For each target t, let $\hat{v}_x^t(k_1+1) = \hat{v}_x^t(k_1)$ and $\hat{v}_y^t(k_1+1) = \hat{v}_y^t(k_1)$.
3. For each k and t, calculate the position of each object as follows:

$$r_x^t(j) = \frac{\sum_{j=k-\min\{k+1,p\}+1}^{\min\{K-p+1,k\}} (\hat{r}_x(j) + (k-j)\hat{v}_x^t\Delta)}{\min\{K-p+1,k\} + \min\{k+1,p\} - k},$$

$$r_y^t(j) = \frac{\sum_{j=k-\min\{k+1,p\}+1}^{\min\{K-p+1,k\}} (\hat{r}_y(j) + (k-j)\hat{v}_y^t\Delta)}{\min\{K-p+1,k\} + \min\{k+1,p\} - k},$$

where Δ is the sampling period.

In addition to the data assignment problem, two computationally serious problems must be addressed to in this problem: one is how to make a full use of a 'window' where linear approximation is valid which we have discussed in the previous subsection 2.1 for maximizing Λ of Equation (1), while perhaps a more serious problem is how we can implement a local optimum criteria without being trapped into many of local minima which we discuss in subsection 5.2 below.

5.1. **Track Estimation .** Exploiting the linearity of approximations allowed within the window, several schemes of approximations may be developed for approximating curved tracks of moving objects in particular as accurate as possible. A simplest scheme is to estimate the $(r_x^t(j), r_y^t(j))$ for all the $j = k_1$ to $k_1 + p - 1$ within the window such that the likelihood function Λ of Equation (1) is maximized, and then go on to the next window having next time index k_2 to $k_2 + p - 1$ without overlapping. In order to obtain a smoother curved track, we can also adopt the following overlapping computational scheme by shifting the window by one time index each time, namely form the window $[k_1, k_1+p-1]$ to $[k_1+1, k_1+p]$, and taking averages of the resulting p values of the position vector $(r_x^t(j), r_y^t(j))$ for each target t. The latter averaging scheme certainly contains redundant computation with added computational costs but is effective in providing a more accurate estimate of a track having larger curvature because an "averaging" process should have a statistical effect of reducing errors. This has been verified by an experimental data of our computation shown in Fig. 2 where the average errors between the estimated positions of targets to their real positions for the curved track marked by A have been reduced by 22.3% from 45.3 to 35.2 between the two methods while for the curve marked by B by 14.8% from 34.4 to 29.3 The averaging process between the shifting window is fully justified due to the independence of the local processing between the windows.

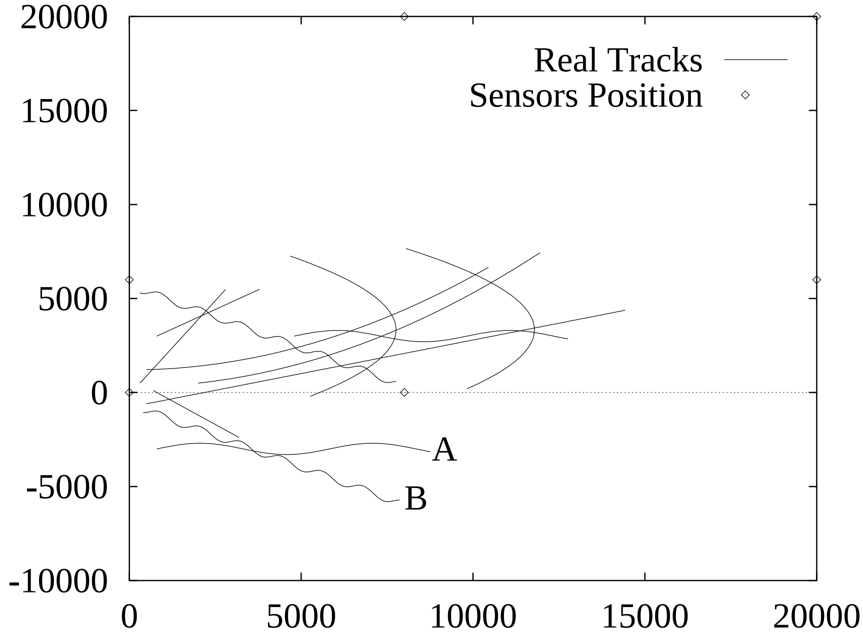
5.2. **Global Minima Criteria .** Many algorithms for the local optimum criteria are suggested, including a stochastic approach such as a simulated annealing method [14], or a most recent SPSA approach [9], or a distributed cooperative method of Yoshida et al [15]. Exploiting the independence of the 2-D data assignment problem, we may resort to a Lagrangian relaxation by releasing constraints [12, 5] associated for the other procedure. The exact working mechanism of these methods is beyond the scope of this paper and the reader may refer to the respective papers for part of the explanations [3, 14, 13, 15, 10]. Because of its simplicity in implementation, we have used the distributed cooperative scheme described in Section 4 for numerical simulations. In this scheme, we form a cluster of independent measuring groups comprising 2 sensors and 1 processor with at least one sensor being overlapped.

Although we may not be able to explain the detailed working mechanism of the distributed cooperative scheme [15] within this paper, a major advantage of the scheme is an extended chance of setting the solution along a optimal path avoiding being trapped into many of local minima by exchanging the initial state vectors of other groups and selecting the group with a minimum errors.

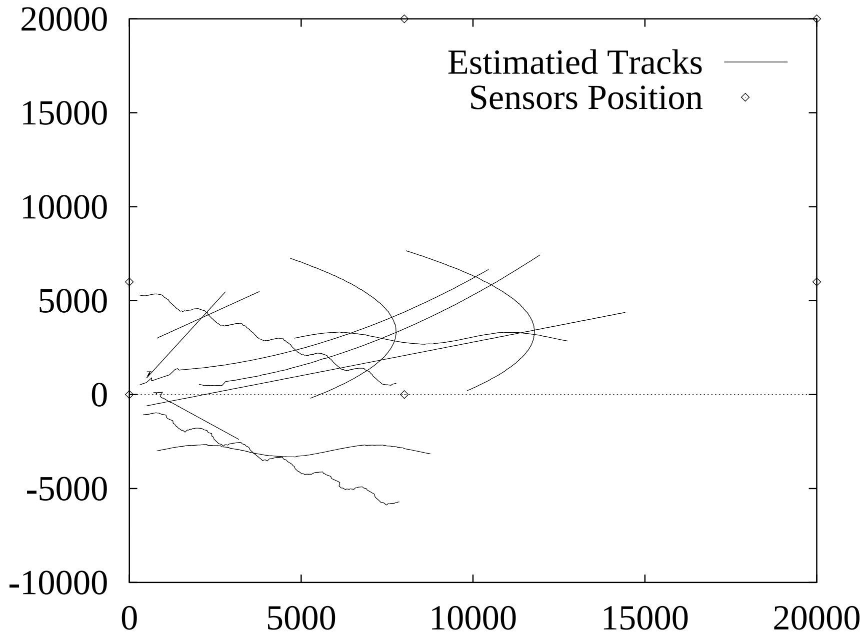
The cooperative process is effective because the minimizing of ASE_G of one group is independent from the other groups, and a best estimation of the initial target state vector as well as the data associations will probably give a closer estimate to the globally optimum solution. Moreover, it is likely that the increasing number of groups increases the chance of success which we can do this by allowing each sensor to belong to more than two groups, but we settle at two in our simulation as a tradeoff between the computing resources.

5.3. **Simulation Results.** In this simulation, we will estimate the tracks of 8 targets using 4 sensors and 12 targets using 6 sensors. Allowing each sensor to belong to two groups with each group having 1 processor, we form 4 groups for 8 targets problem and 6 groups for 12 targets problem. In the simulation we use 200 measuring samples where each of the successive 4 data samples captured by a window can be regarded as a linear portion. It is to implement the minimization of ASE that we have overlapped the computation by shifting the window by one time index instead of 4 time indexes. The standard deviations of white Gaussian noise are set to $0.2°$ for each sensor. In the simulation, we used one 400MHz Pentium II computer, and the following convergence criteria are used; the minimum $\mathrm{ASE}_G < 0.3$ or the two successive difference of minimum $\mathrm{ASE}_G < 0.01$ subject to the additional condition of minimum $\mathrm{ASE}_G < 1.0$. Some hundreds of experiments are done in the presence of randomly added Gaussian noise keeping the required deviation level to real bearing data. A typical result of simulations is given in Fig. 2 where the final average of the minimum ASE_G is 0.61, showing an excellent agreement.

It is likely that the computing time spent on the data assignment procedure does not constitute the major part of the procedure for the multitarget tracking problem; in fact in our simulation of the 8 targets and 12 targets problems, the total computing time costs are of order of 1 minutes while the time spent on data assignment problem is of order of 1 seconds implying that it constitutes less then 2% of the total computing time. The advantage of our $O(N \log N)$ sorting based algorithm over the $O(N^3)$ Hungarian type algorithms becomes evident only for sufficiently large N, where the problem of local optima becomes more serious. In fact, a better scheme for a local optimal solution is independently needed.



(1)



(2)

FIGURE 2. The Real Tracks and The Estimated Tracks for 12 Targets

6. **Conclusion .** In the previous sections, we have presented a distributed cooperative computational method for the multi-target tracking problem.

Compared with the traditional centralized computing scheme which makes use of the standard or improved Hungarian assignment algorithm in data assignment problem, our simulations show that our computational method could reduce the computational complexity quite drastically in agreement with the prediction of [14].The key is the theorem we have proved in this paper which shows that the sorted bearing measurement data provides an optimal data assignment matrix requiring only the sorting complexity of $O(N \log N)$.

As in all other existing works on the subject [14, 8, 10, 16], we have formulated the ML-based relaxation method exploiting a minimal value in the square errors between estimated and observed bearing measurements. Taking full advantage of distributed cooperative computing scheme where multiple processors cooperate to obtain a global minimum, we are able to compute 12 target motion analysis with variable directions and speeds within 1 minute of single 400MHZ Dell computer time. The solution we found seems to be *suboptimal* in the sense of Ting & Iltis [14].

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